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Original Article

Bayesian regression for capital asset pricing model

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ABSTRACT: In this paper, we critically evaluate the Capital Asset Pricing Model (CAPM) and its limitations in predicting future returns using Linear Regression (LR) models. We propose an alternative approach, Bayesian Regression, which offers a more informative and accurate prediction framework. Our study compares the performance of LR and Bayesian Regression models in forecasting the returns of popular cryptocurrencies, Doge (for asset) and Bitcoin (for market). Through the use of Mean Squared Error (MSE), we demonstrate that the Bayesian Regression model outperforms the LR model in terms of prediction accuracy. The findings highlight the advantages of Bayesian methods in capturing the complex relationships and uncertainties inherent in financial markets. Our research contributes to the ongoing discourse on investment decision-making, providing valuable insights into the effectiveness of Bayesian Regression in the context of cryptocurrency investments.

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1. Introduction

The Capital Asset Pricing Model (CAPM), initially proposed by William Sharpe in 1964 [13] and later refined by John Lintner in 1975 [8], transformed the landscape of finance by offering a systematic approach to understanding the relationship between risk and expected return. This model has become a cornerstone in finance education, widely utilized for estimating the cost of capital and assessing portfolio performance. Sharpe's pioneering contributions to the CAPM earned him the Nobel Memorial Prize in Economic Sciences in 1990.

The CAPM's lasting relevance is reflected in its continuous use by investors, finance professionals, and academics. It has endured through changing market dynamics, shaping our comprehension of financial systems and influencing investment strategies. Despite the development of alternative models, the CAPM remains a vital instrument for elucidating the dynamics of risk and reward across various financial scenarios.

However, the CAPM is not without its criticisms, particularly concerning its empirical limitations. This paper aims to explore these shortcomings through the application of Bayesian Regression to the CAPM framework.

Building on the foundational work of Harry Markowitz, who introduced the concept of portfolio selection in 1959 [10], the CAPM extends the "mean-variance model" that addresses the preferences of risk-averse investors.

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Markowitz's model emphasizes the importance of maximizing expected returns while minimizing risk variance, setting the stage for the CAPM's analysis of the risk-return trade-off.

The CAPM is predicated on two central assumptions: the existence of a risk-free rate and the concept of a market portfolio. These elements allow investors to gauge their portfolios' expected returns and risks in relation to the broader market, facilitating more informed investment decisions and deepening the understanding of the risk-return relationship.

Moreover, the CAPM builds upon Markowitz's framework by delineating conditions for mean-variance-efficient portfolios [10]. It presupposes a consensus among investors regarding the distribution of asset returns, along with the capacity to borrow and lend at a risk-free rate. Such assumptions enable the exploration of the risk-return relationship, helping to identify potentially mispriced portfolios and uncover opportunities for achieving abnormal returns.

2. Why we use Bayesian Regression

Bayesian Regression is a powerful statistical technique that offers several advantages over traditional regression methods.Bayesian Regression offers a robust and flexible approach to statistical modeling, providing a comprehensive understanding of uncertainty, incorporating prior knowledge, handling complex models, and making probabilistic predictions. It is particularly valuable in situations where uncertainty quantification, model comparison, and the incorporation of prior information are important considerations [4, 1, 11, 9]. Jorion [6], Introduced an empirical Bayes estimator for expected returns, shrinking individual asset estimates toward a grand mean, reducing estimation error. Pastor and Stambaugh [12], Used an empirical Bayes framework to incorporate prior beliefs about factor pricing, enhancing predictive performance in portfolio optimization. Dumas et al. (2003) – Applied Bayesian techniques to estimate global CAPM with country-specific shocks, finding stronger predictability in emerging economies.

Here are some reasons why Bayesian Regression is used:

1. Uncertainty Quantification: Bayesian Regression provides a comprehensive understanding of uncertainty by quantifying it in a probabilistic manner. It estimates the probability distribution of model parameters rather than just providing point estimates. This allows for more informed decision-making, as it captures the range of plausible values for the parameters and their associated uncertainties.

2. Prior Information Incorporation: Bayesian Regression allows the incorporation of prior knowledge or beliefs about the model parameters. Prior distributions can be specified based on expert opinions, historical data, or domain-specific information. This feature is particularly useful when there is limited data or when certain parameters are known to have specific characteristics (e.g., non-negative values or specific ranges).

3. Regularization and Model Complexity: Bayesian Regression naturally handles model complexity and overfitting through the use of priors. By choosing appropriate prior distributions, the model can regularize the parameter estimates, preventing them from becoming too sensitive to noise in the data. This helps strike a balance between model complexity and the available data, improving predictive performance.

4. Small Sample Size and Missing Data: Bayesian methods excel in situations with small sample sizes or missing data. The use of prior distributions can compensate for limited data, providing more robust estimates and predictions. Bayesian techniques, such as Markov Chain Monte Carlo (MCMC) methods, can handle missing data elegantly by integrating over the missing values during inference.

5. Probabilistic Predictions: Bayesian Regression provides probabilistic predictions rather than just point estimates. It produces predictive distributions, which include information about the uncertainty associated with each prediction. This is especially valuable in decision-making scenarios where understanding the range of possible outcomes is crucial.

6. Model Comparison and Selection: Bayesian Regression facilitates model comparison and selection by providing a natural framework for model averaging and model selection. The posterior probabilities of different models can be calculated, allowing for a quantitative assessment of their relative plausibility. This helps identify the most suitable model for the given data and problem at hand.

7. Sequential Learning and Adaptation: Bayesian methods are well-suited for sequential learning and adaptive modeling. As new data becomes available, the posterior distribution from the previous analysis can be used as the prior for the updated analysis. This enables the model to learn and adapt over time, incorporating new information seamlessly.

8. Customizable and Flexible: Bayesian Regression offers flexibility in modeling complex relationships. It can handle non-linear relationships, hierarchical structures, and mixed-effects models. By specifying appropriate prior distributions and likelihood functions, Bayesian Regression can be tailored to fit a wide range of data and modeling scenarios.

9. Interpretability: Bayesian Regression provides a more nuanced interpretation of model parameters. The posterior distributions capture the uncertainty around parameter estimates, allowing for a more comprehensive understanding of their credibility and precision. This can aid in making informed decisions and interpreting the impact of variables in a probabilistic context.

10. Objective Bayesian Inference: Bayesian Regression can be used with objective priors, which are derived from the data or the structure of the problem itself. Objective Bayesian methods aim to provide inference that is independent of subjective prior beliefs, making the analysis more objective and reproducible.

Black and Litterman [2] introduced the famous Black-Litterman model, which uses Bayesian techniques to combine prior beliefs with market equilibrium conditions to optimize a global investment portfolio. In Chapter 12 of this book [7] provides a detailed exposition of Bayesian methods for estimating CAPM models, including shrinkage estimation, hierarchical modeling, and Markov Chain Monte Carlo methods.

3. CAPM and BCAMP

General formula of CAPM is as follow:

$$r_i = r_f + \beta(R_M - r_f) + e_i, \tag{1}$$

where

1. r_i: return of i^{th} asset
2. r_f: rate of risk free asset
3. R_M: return of market
4. e_i: is white noise with mean 0 and variance σ^2

The $(R_M - r_f)$ is risk premium and ordinary least squared (OLS) estimation of beta is $\beta = \frac{cov(r_i, R_M)}{var(R_M)}$. So if we take mathematical expectation both side 1 we have;

$$E(r_i) = r_f + \beta(E(R_M) - r_f).$$

Fama and French [3] mentioned that the original version of the Capital Asset Pricing Model developed by [13] and [8] has not been successful in empirical testing.

We employed a contemporary adaptation of the Capital Asset Pricing Model (CAPM), known as the Bayesian Capital Asset Pricing Model (BCAPM), to assess the performance of our investments. The BCAPM incorporates a Prior distribution of coefficients, leveraging Bayesian principles to enhance the estimation of expected returns. This innovative approach considers unique risks and characteristics within specific business areas, such as production, marketing, and finance. By accounting for prior knowledge, the BCAPM enables more precise evaluations and empowers organizations to make well-informed investment decisions. The general formula for the BCAPM is as follows:

$$r_i = \alpha + \beta(R_M) + e_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

that

$$e_i \overset{iid}{\sim} N(0, \sigma^2),$$

with OLS estimation we have:

$$\hat{\sigma^2} = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y_i})^2,$$
$$\hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y_i})}{\sum_{i=1}^{n}(x_i - \bar{x})^2},$$
$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x},$$
$$Var(\hat{\alpha}) = \hat{\sigma^2}\left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right),$$
$$Var(\hat{\beta}) = \frac{\hat{\sigma^2}}{\sum_{i=1}^{n}(x_i - \bar{x})^2}.$$

In this paper used Informative Prior distribution as follows [5]:

$$\alpha|data \sim N(\alpha_0, S_\alpha), \tag{3}$$

and

$$\beta|data \sim N(\beta_0, S_\beta), \tag{4}$$

$$\frac{1}{\sigma^2} \sim Gamma(\frac{\nu_0}{2}, \frac{\nu_0\sigma_0}{2}), \tag{5}$$

that $\alpha_0, S_\alpha, \beta_0, S_\beta, \nu_0, \sigma_0$ are hyperparameters [5] and estimated numerically and by using MCMC methods which in this paper computed by $stan\_glm$ function in R4.4.0 programming.

4. Comparison CAPM and BCAPM with real data

For analyzing of real data, we consider daily historical data of $DOGE$'s log-returns for asset return and $BITCOIN$'s log-returns for market return ($r_t = (log(P_{t+1}) - log(P_t))$ that r_t is log-return and P_t is price in t^{th} day). we assume the absence of a risk-free asset. Data are from 2018-01-01 till 2023-12-01 and downloaded from Yahoo Finance.

As depicted in Figure 1, the historical price and logarithm of return are displayed.

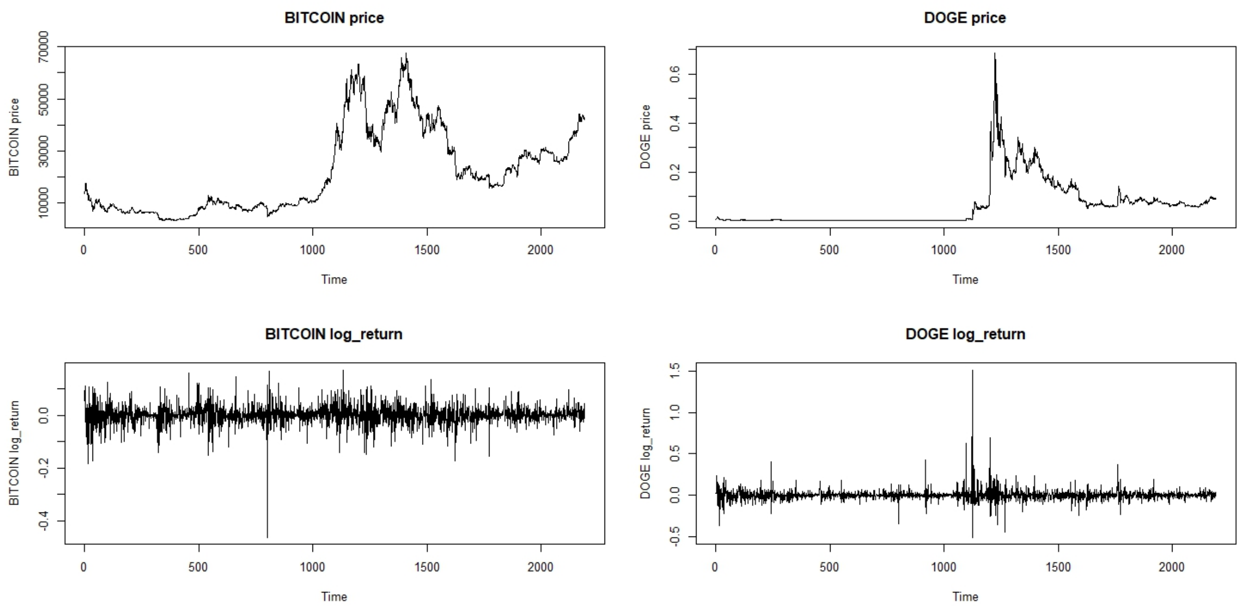

Figure 1: plot historical data of $BITCION$ and $DOGE$

4.1. CAPM

For CAPM we used simple Linear Regression that results is as follow;

$$r_i = \beta_0 + \beta_1 R_M + e_i \tag{6}$$

The result for CAPM is

Table 1: Regression coefficient estimation

Coefficients	Estimate	$p-$value
Intercept	0.0007	0.64
return Bitcoin	0.998	0.00

Based on the output in Table 1, we can see that at a significance level of 0.05, the null hypothesis for the intercept term is accepted, So best model is

$$\hat{r}_i = 0.989 R_M. \tag{7}$$

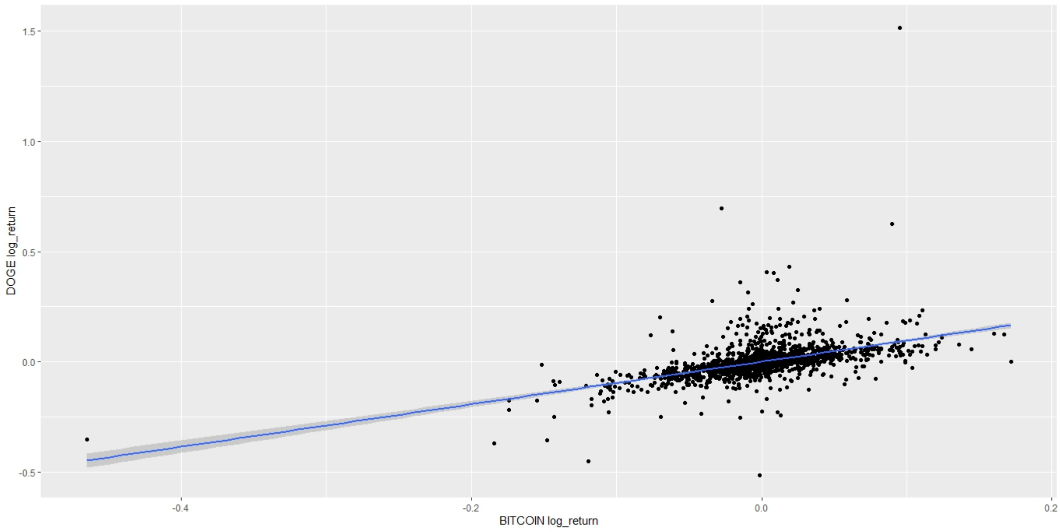

Figure 2: plot log-return of *DOGE* and *BITCOIN* with Linear Regression Line

The points that are farthest away from the regression line in Figure 2 have the highest residuals or errors, which can result in a higher mean squared error (MSE) value. This indicates that these points are not well-explained by the regression model.

4.2. BCAPM

For BCAPM we used Bayesian Regression that results is as follow;

$$r_i = \alpha^B + \beta^B(R_M) + e_i. \tag{8}$$

The results are:

Table 2: Bayesian Regression coefficient estimation

Coefficients	Estimate	95% confidence Interval
Intercept	0.005	(0.0012,0.0088)
return Bitcoin	1.123	(0.881, 1.365)

Based on the output in Table 2, we can see that

$$\hat{r}_i = 1.123R_M + 0.005. \tag{9}$$

For Model (9), the Shapiro-Wilk test results (p = 0.07) and the Q-Q plot presented in Figure 3 suggest that the normality assumption is violated.

4.3. Comparison MSE of CAPM and BCAPM

To evaluate the predictive accuracy of both models, k-fold cross-validation was employed, and the Mean Squared Error (MSE) was calculated for each. Table 3 presents the MSE values obtained for the Bayesian Regression model and the classical Linear Regression model. As indicated in the table, the Bayesian Regression model exhibits a lower MSE, suggesting that it outperforms the classical Linear Regression model in terms of prediction accuracy. The lower MSE value demonstrates the Bayesian model's superior ability to minimize the average squared difference between predicted and actual values.

Table 3: Average of MSE and MAE of 10-fold cross validation

Model	MSE	MAE	R^2
Linear Regression	0.00287	0.0263	0.51
Bayesian Regression	0.00198	0.0211	0.63

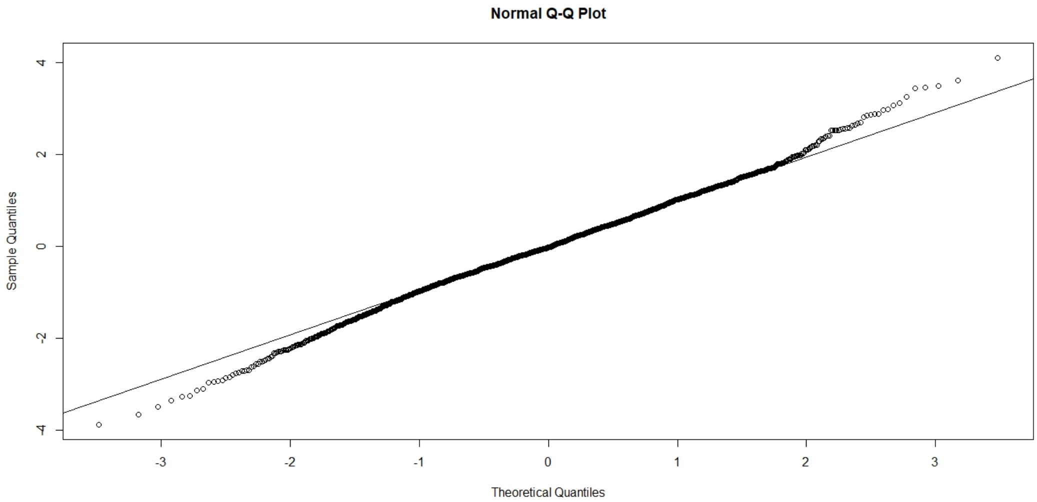

Figure 3: Q-Q plot of model 9

5. Future works

For future endeavors, we propose exploring novel Bayesian methods, including:

1. Using Reference Prior such as Jeffreys Prior
2. Using heavy tail distribution such as T-student or Cauchy distributions
3. In Linear Regression, the estimation of coefficients is inherently dependent. To account for this dependency, a bivariate normal distribution is utilized as the prior distribution for the coefficients.

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