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Original Article

# Systemic risk in financial networks with two central institutions

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**ABSTRACT:** Systemic risk in the interbank market is the topic of this article. This market is modeled as a directed graph, where the edges are the bank-to-bank liabilities and bank-to-end users liabilities and the nodes are the banks. Our study extends the modeling paradigm of Amini et al. [3] by adding a second Central node to the system and using the equilibrium equation of the Veraart et al. [11] with some modifications that are better suited to our model. We study the effects of two central nodes on a financial network. It is evident that two central nodes can reduce the end-users shortfall and increase the predicted surplus of the banks when compared to a single central node. We provide a few straightforward examples to demonstrate our findings.

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## 1. Introduction

The use of central clearing has emerged as a crucial aspect of the global derivatives markets subsequent to the Global Financial Crisis. Central nodes (CNs) are positioned at the center of centrally cleared markets, where mandates to centrally clear derivatives have fundamentally changed the structure of financial networks. The Asian financial crisis in the late 1990s and the more recent banking crisis in 2007–2008 have demonstrated how susceptible the banking system may be in general to asset degradation, liquidity problems, and a decline in confidence. Due to the urgency of the crisis, the majority of modeling work done thus far has been on static models. Conventional remedies, like issuing fresh bonds or shares, cannot be completed in the short timeframes that are now accessible, usually a few days. The market's structure of intermediation is altered by the introduction of a central node (CN), which acts as an intermediary for any financial obligations between banks. CNs are capitalized with equity, a guarantee fund, and a fee from end users that takes the form of a bank's participation in the guarantee fund, which is paid to the

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second central node to give them priority in the event that the banks fail, after achieving equilibrium and granting rights to owners, the remaining fee will be returned to the end user, without the central bank receiving anything in return. This encourages end users to participate in the system and ensures their rights are better protected. This means that, we have two central banks, two guarantee funds, and two central bank capitals that are used in the event that a bank has a shortfall. The presence of two central banks may give the impression that the system is complex, however, if we refer back to the first reference, which examined only one central bank, we discover that our system is not complicated at all, as we have integrated the objectives of the two banks and utilized a single variable in the equilibrium equation to account for both of them. Moreover, our system is more straight forward when we refer back to the second study, which examined two or three Central banks. Our suggested CNs architecture lowers shortfall losses and bank liquidation costs while increasing aggregate surplus. We describe the equity, fee, and guarantee fund policies of the CNs that lower systemic risk and align with bank incentives. Disputes arise about the specifics of the CNs arrangement, especially those pertaining to capitalization. In fact, in the event of a clearing member's default, the losses are initially covered by the member's own endowment, equity, and guarantee fund payments made to the CN, as well as by the fee collected from end users, and lastly by the CN's own equity. It demonstrates that having two central nodes means that the capital of the banks may be cleared more effectively since the bank deficit is decreased. For this reason, we provide guidelines for choosing the external capital of CNs and the contributions to the guaranty fund. The early liquidation losses are a crucial component of the model: banks are random endowment $Qi \geq 0$ of a nonpledgeable asset, which are liquidated at a loss when the banks settle their liabilities.In our work, the CN is not intended to be a representation of a real clearinghouse. As a matter of fact, the contributions of members are divided by the clearinghouses of today into three distinct layers: variation margin, initial margin, and default fund. The initial margin and the default fund are the crucial layers in the case of a severe occurrence. When the initial margin is removed, our guarantee fund is most similar to a default fund. The guaranty fund's bank shares are combined and used to cover member losses. It is anticipated that members would first deplete their personal contribution to the guaranty fund before any leftover losses trickle down to the pooled shares of the guaranty fund held by the other members. Morally, the default fund plus the initial margin of a real-world CCP should be more than the guaranty fund of our model since the initial margin is absent from our model. In our approach, we have profited from the first portion of the Veraart et al. [11] equilibrium equation making some modifications to make it consistent with Amini et al. [3] and introducing a second central bank for end users, which allows central banks to come up with a more practical plan for banks that have failed to make improved loan repayments. We have required the capital of the central nodes to be equal to or greater than the average member capital of our system. Our research is founded on the Amini et al. [3], which has focused on the OTC market. While choosing our graph, we took into consideration a Veraart et al. [11] that examined the probability of bank loans at random. We looked at the worst-case scenario in this research and found that having two central banks would greatly reduce risk for both banks and the end users.

*Connection to earlier works of literature.*

Amini et al. [3] elucidate various aspects of centralized clearance, arguing that a central node (CN) minimizes bank liquidation and improves aggregate surplus. The impact on individual bank surplus is uncertain and hinges on the trade-off between CN deficiency and reduced bank counterparty risk from an ex-ante perspective.

Eisenberg and Noe [7] present the requirements for the existence and uniqueness of a clearing vector in complex financial systems, examine the characteristics of the clearing vector, and offer comparative statistics that illustrate how the financial system's underlying parameters and the clearing vector relate to one another.

Duffie and Zhu [6] demonstrated how netting efficiency may be reduced by segregating the central clearing of a specific class of derivatives, such as credit default swaps, which would increase predicted counterparty exposures and collateral requirements. It is more effective to clear several classes of derivatives on the same central counterparty (CCP) rather than separate CCPs. In our study, two central banks collaborate to achieve a common goal, as opposed to operating separately.

According to Cifuentes et al. [4], capital buffers and liquidity buffers serve similar purposes. In some circumstances, liquidity constraints may be a more effective means of preventing systemic effects than capital buffers. Requirements for liquidity may mitigate some of the externalities caused by the price impact of selling into a declining market.

Rogers and Veraart [10] analyzed how bank consortiums could potentially save failing banks. This analysis led to crucial conclusions that any bank consortium with the means to do so will be motivated to act, and any consortium that would fail if default spread will be motivated to rescue if it has the means. Although these results are encouraging, they do not guarantee the rescue of failing institutions.

Amini et al. [2] demonstrated that the impact of forced liquidations on prices and shortfalls is reduced with full multilateral netting.

Amini et al. [1] clarify several issues related to central counterparty clearing in their theoretical study. A CCP increases aggregate surplus by reducing banks' deficits and liquidation losses, and sets up adequate parameters to decrease systemic risk concerning the CCP's guarantee fund and fee policies.

Glasserman et al. [9] argue that information about positions inside CCPs must be shared among CCPs and clearing members. If sharing is not feasible due to the sensitive nature of the data, each CCP should cautiously assume the positions that clearing members hold at other CCPs (with a correspondingly conservative margin charge) and offer clearing members a potential margin reduction as an incentive to provide this information. A CCP may choose to assume conservatively by comparing the positions it clears with the total number of outstanding positions for all parties involved, including other CCPs. The Dodd-Frank Act mandates swap data repositories to collect this kind of aggregate data.

## 2. Financial Network

We examine a set of $m$ interconnected financial organizations(bank) with $i = 1, \ldots, m$. A two-round clearing technique is proposed by Ghamami et al. [8] for collateralized markets. Who defaults and how much is initially paid to counterparties are decided upon in the first round. If the first round's payments weren't sufficient, the collateral (initial margin) that wasn't used in that round is given back to the nodes that set it aside for that purpose in the second round [11] extended the first round of clearing to account for exogenous default costs and other market frictions, as well as varying degrees of fluctuation in margin gains that are trimmed by CCPs. The second round of clearing, as suggested by [8], will then be employed.

We utilize three time periods, Values at date $t = 0$ are deterministic and values at date $t = 1, 2$ are random variables on a probability space $(\Omega, F, P)$ with $\Omega$ a set of extreme scenarios that realize at time 1. Liabilities $P_{ij} = P_{ij}(\omega)$ represent claims that become due at date 1 given an extreme scenario $\omega \in \Omega$. At the initial round $t = 0$ bank $i$ holds $\gamma_i \geq 0$ units of a liquid asset (cash), with zero return. Values at the first round date $t = 1$, banks receive a random endowment $Q_i \geq 0$ of a nonpledgeable asset(illiquid asset). In addition to identifying the banks that are in danger of failing, The asset can be liquidated at its fundamental value $Q_i$ only at the second round date $t = 2$ (e.g., the long-term investment matures at $t = 2$). If the asset is liquidated at $t = 1$ only a fraction of its fundamental value can be recovered, since early liquidation is costly. Instead of fixed early liquidation value we can use variation margin gains haircutting (VMGH)(When it comes to risk management of the defaulters positions, haircutting VM gains incentivizes all clearing participants to bid aggressively in default management auctions and supply hedges to the banks that are in short supply. Furthermore, no participant experiences an uncontrollable loss of earnings even when losses are practically uncappable. This prevents uncontrollable losses from being concentrated on any one type of clearing participant.) [5] and applying the first component of [11] equilibrium equation's and exchange bank loans with one another using this method.

We denote by $\varphi_i$ the liquidation value of the asset at $t = 1$ and we assume that $\varphi_i \leq Q_i$ if $Q_i > 0$ (naturally, $\varphi_i = 0$ if $Q_i = 0$). It is great if we can pay off all of the bank obligations; however, if that is not feasible, we will turn to central banks and their guarantee funds.

Given the extreme circumstances under consideration, it is logical to infer that banks cannot use the asset as collateral for borrowing, thereby rendering it nonpledgeable. Under these conditions, financial liquidity is not available unless a lender of last resort offers it, which is not currently being considered. The systemic risk literature widely accepts both assumptions. Furthermore, banks have obligations to end users in addition to interbank responsibilities. If there are any liabilities from end users to the banks to better safeguard their rights, we establish a second central bank (where the two central banks work together to lower the risk to end users and banks and increase the overall surplus to the network). This second central bank will be devoted to end users, and a second guarantee fund(fee)is made for it; the fee is paid by the end user so that, in the event that the banks fail or go broke, the first central bank, with the assistance of the second central bank, gives priority to end users in granting their rights based on the capital they possess.

*Nominal interbank liabilities*

The total nominal interbank obligation, realized at time 1, are represented by a nonnegative random matrix $P_i$,where $P_{ij} \geq 0$ with $P_{ii} = 0$ denotes the cash amount that bank $i$ owes bank $j$ at $t = 1$. The liability of bank $i$ to the end users is denoted by $D_i \geq 0$. The total nominal interbank liabilities of bank $i$ sum up to

$$P_i = \sum_{i=1}^{m} P_{ij}.$$

Bank $i$ in turn claims a total nominal cash amount of $\sum_{j=1}^{m} P_{ji}$ from the other banks according to the method of illiquid nonpledgeable asset that illustrate in the clearing equilibrium equation. The nominal balance sheet of bank $i$ at $t = 1$ is given by

- Assets: $\gamma_i + \sum_{i=1}^{m} P_{ji} + Q_i$;
- Liabilities: $P_i + D_i + nominal\ networth.$

The nominal cash balance is

$$\Lambda_i = \gamma_i + \sum_{j=1}^{m} P_{ji} - (P_i + D_i).$$

*Central counterparty clearing*

We extend the preceding setting by adding a second CN to the financial network. We formally label it as entity $i = 0$, $\gamma_0 \geq \sum_{i=1}^{m} \frac{\gamma_i}{m}$ for both central banks (That's why we choose one symbol for the two central banks because in reality we use the capital of both central banks for same purpose). We assume that all interbank liabilities among banks (but not toward end users) are cleared through the CNs. The CNs is capitalized in cash with equity $\gamma_0$ for $CN_{1,2}$ and the banks transfer cash at time 0 to the CN. The guarantee fund contribution will be denoted by $g_i$. It is a cross between the initial margin and the default fund contribution of the real-world CCP. As an advance payment, $\sum_{i=1}^{m} g_i$ is to be netted against the liabilities from each bank $i$ to the CN, just as a margin of a real-world CCP. And $\sum_{i=1}^{n} f_i$ which is funded by up-front cash-contribution $f_i \leq D_i$ from every end users $i$ ($D_i$ is the capital that the end users put in the banks).

We permit netting bank $i$'s nominal obligation to the CN against the upfront guarantee payment $g_i$:

$$P_{i0} = (\Lambda_i + g_i)^- .$$

Note that $P_{i0}$ is positive if and only if $(\Lambda_i)^-$ exceeds $g_i$.

The total nominal liability of the CN equals

$$P_0 = \sum_{i=1}^{m} P_{0i} = P_i + D_i.$$

*Nominal guaranty fund.*

We define the nominal share of bank $i$ in the guaranty fund as

$$G_i = (\Lambda_i + \ g_i)^+ - \Lambda_i^+ = \begin{cases} g_i & \text{if } \Lambda_i > 0, \\ g_i + \Lambda_i & \text{if } -g_i < \Lambda_i \leq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

This is dependent on $\Lambda_i$ being realized.Consequently, we have $G_i - P_{i0} = g_i - \Lambda_i^-$, and $G_i \times P_{i0} = 0$.

We indicate by $G_{tot} = \sum_{i=1}^{m} G_i$ the whole nominal amount of the guarantee fund. The nominal balance sheet of the CN at t=1 becomes

- Assets: $\gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} f_i + \sum_{i=1}^{m} P_{i0}$;
- Liabilities: $P_0 + G_{tot} + nominal\ networth.$

**Remark 2.1.** *At $t = 2$, banks repay their remaining guarantee fund shares.*

The member's entitlement to use the mutualized portion of the guarantee fund to offset his personal obligation in the case of an extreme circumstance remains intact. It is obvious that the definition of "netting with a guaranty fund" has to be clarified in the context of our two-period model. Our approach represents a single "armageddon" realization of liabilities rather than daily margining with a CN. Although it is not something we recommend, members may net their daily liabilities against the posted guarantee fund in an emergency. In actuality, following an extraordinary incident, a new guaranty fund has to be established if a member nets their contribution to the fund with their liabilities. Older members who are unable to contribute to the guarantee fund may be removed. The post-extreme event CN recovery (including guarantee fund replenishment) surpasses the capabilities of our methodology.

*Interbank liability clearing equilibrium.*

As we meantioned before, the payables to the end users are senior with respect to the interbank payables. We let the cash, net of outside payables, be

$$\Gamma_i = \gamma_i - D_i.$$

The first part of interbank liability clearing equilibrium is if bank $i$'s cash balance is negative, $\Gamma_i + \sum_{j=1}^{m} P_{ji} < P_i$, then bank $i$ can use variation margin gains haircutting (VMGH) because has a liquidity shortfall and is forced to sell its illiquid asset by using $\varphi_i = Z_i \wedge 1$, where $Z_i = \exp(-\alpha Q_i)$.
The parameter $\alpha \geq 0$ is used to model the price impact of illiquid nonpledgeable asset $Q_i$. If $\alpha = 0$, then there is no price impact and $\varphi_i = 1$. If $\alpha > 0$ then $\varphi_i < 1$, hence capturing the decline in the price of nonpledgeable asset when the share $Q_i$ of nonpledgeable asset is sold.

As we explain better in the definition below, the amount of $\sum_{j=1}^{m} P_{ji}$ according to the amount $Z_i$ will be taken from banks.

If the revenue from the illiquid does not cover the negative cash balance, $\Gamma_i + \sum_{j=1}^{m} P_{ji} + \varphi < P_i$, then bank $i$ is in default, then it's asset is liquidated in full, $\varphi_i = 1$. After that, we have to do the other steps of clearing equilibrium equation.
The clearing payment of bank $i$ to the CNs is

$$P_i^* = P_{i0} \wedge (\gamma_i - g_i + \varphi_i)$$

The value of the CNs's total assets becomes

$$A_0 = \gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} f_i + P_i^*. \tag{2}$$

After the payable to end users, the clearing interbank liability payment of the CN to bank $i$ is defined according to the proportionality rule $P_0^* \times \Pi_{0i}$. As explained in the following definition which is the last part of Interbank liability clearing equilibrium.

**Definition 2.2.**

$$P_0^* = \begin{cases} Case_1 : P_i + D_i & if P_0^* = P_0 \\ Case_2 : D_i^* = D_i \wedge (A_0 + \sum_{j=1}^{m} \ell_i \pi_{ji}) \quad afterthat, & \\ ((A_0 + \sum_{j=1}^{m} \ell_i \pi_{ji}) - D_i)^+ \quad then \quad \Pi_{0i} = \frac{P_{ij}}{P_0^*} & if P_0^* < P_0 \end{cases} \tag{3}$$

As we explain in the example, the amount of $\sum_{j=1}^{m} P_{ji}$ that bank $i$ in turn claims a total nominal cash amount from the other banks is taken by $\sum_{j=1}^{m} \ell_i \pi_{ji})$, according to the amount of $Z_i = exp(-\alpha Q_i)$, i.e, we can define the shorfall payable between banks as

$$S = \sum (\sum_{j=1}^{m} P_{ji} - \sum_{j=1}^{m} P_{ji}(Z_i)).$$

*Liquidation of the guaranty fund.*

We now determine the clearing payments from the guaranty fund to the banks at $t = 2$. With "senior CN," the guaranty fund is the first layer to absorb shortfall losses imposed by defaulted banks, prior to the nominal net worth of the CNs. The guaranty fund's surplus in the clearing equilibrium is given by

$$G_{tot}^* = G_{tot} \wedge (A_0 - P_0 - \gamma_0)^+. \tag{4}$$

The clearing payment from the guaranty fund to bank $i$ is defined by the proportionality rule,

$$G_i^* = \frac{G_i}{G_{tot}} \wedge G_{tot}^* (= 0 \quad if \quad G_{tot} = 0.)$$

This means that banks' shares absorb losses in the guaranty fund proportionally.

*Terminal net worth.*

The net worth of the CN at $t = 2$ is defined by

$$C_0 = A_0 - P_0 - G_{tot}^*. \tag{5}$$

The shortfall of the CNs becomes

$$C_0^- = (A_0 - P_0)^-. \tag{6}$$

The value of bank $i$'s asset, including its share in the guaranty fund, at $t = 2$ becomes

$$A_i = \Gamma_i + \varphi_i Z_i + (1 - \varphi_i)Q_i + P_0^* \times \Pi_{0i} + G_i^* - g_i. \tag{7}$$

The net worth of bank $i$ at $t = 2$ is defined by

$$C_i = A_i - P_{i0}. \tag{8}$$

The totall shortfall of bank $i$ equals

$$C_i^- = (A_i - P_{i0})^- = (D_i - D_i^*) + (P_{i0} - P_i^*). \tag{9}$$

We denote that the shortfall imposed by $D_i^- = D_i - D_i^* = C_i^- - (P_{i0} - P_i^*)$. Thus $D_i = D_i^* + C_i^- - (P_{i0} - P_i^*)$. Also, we can write in the form

$$-D_i - (P_{i0} - P_i^*) = -D_i^* - C_i^-. \tag{10}$$

The pertinent number we will take into account is $C_i^+$ from the standpoint of the bank's utility. The end users total receivables $\sum_{i=1}^m D_i^*$ indicate their total utility.

## 3. Result

*Aggregate surplus identity with CN*

Now, we prove the aggregate surplus identity with CN.

**Lemma 3.1.** *The total excess with CN equals*

$$\sum_{i=0}^m C_i^+ + \sum_{i=1}^m D_i^* = \sum_{i=1}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \varphi_i(Q_i - Z_i).$$

*Therefore, the clearing mechanism is the only way that the aggregate surplus is dependent on the implied liquidation losses.*

**Proof.** We have that

$$\sum_{i=0}^m C_i = C_0 + \sum_{i=1}^m C_i.$$

From (5) and (8), we obtain

$$\sum_{i=0}^m C_i = A_0 - P_0 - G_{tot}^* + \sum_{i=0}^m A_i - \sum_{i=0}^m P_{i0}.$$

By (2) and (7), we get

$$\sum_{i=0}^m C_i = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m f_i + P_i^* - P_0 - G_{tot}^* + \sum_{i=1}^m \Gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \varphi_i(Q_i - Z_i) + P_0^* \times \Pi_{0i} + G_i^* - g_i - P_{i0},$$

which implies

$$\sum_{i=0}^m C_i = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m f_i + P_i^* - P_0 - G_{tot}^* + \sum_{i=1}^m \gamma_i - \sum_{i=1}^m D_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \varphi_i(Q_i - Z_i) + P_0^* \times \Pi_{0i} + G_i^* - g_i - P_{i0}.$$

Therefore

$$\sum_{i=0}^m C_i = \sum_{i=0}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \varphi_i(Q_i - Z_i) - (P_0 - P_0^*) - \sum_{i=1}^m D_i - \sum_{i=0}^m (P_{i0} - P_i^*).$$

In the previous equality, we used the assumption that $\sum_{i=1}^m \Pi_{0i} = 1$.
And where we used (10) we obtain

$$\sum_{i=0}^m C_i = \sum_{i=0}^m \gamma_i + \sum_{i=1}^m Q_i - \sum_{i=1}^m \varphi_i(Q_i - Z_i) - (P_0 - P_0^*) - \sum_{i=1}^m D_i^* - \sum_{i=1}^m C_i^-.$$

By reordering the terms,

$$\sum_{i=0}^{m} C_i + \sum_{i=1}^{m} C_i^- + \sum_{i=1}^{m} D_i^* = \sum_{i=0}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} \varphi_i(Q_i - Z_i).$$

Hence,

$$\sum_{i=0}^{m} C_i^+ + \sum_{i=1}^{m} D_i^* = \sum_{i=0}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} \varphi_i(Q_i - Z_i).$$

□

**Example 3.1.** *In this example, we explain that having two central banks for end-users and banks is better than having a central bank or not having a central bank. We assume that* $\gamma_1 = 3, \gamma_2 = 2, \gamma_3 = 1, \sum_{i=1}^{3} g_i = 3$ *(The*

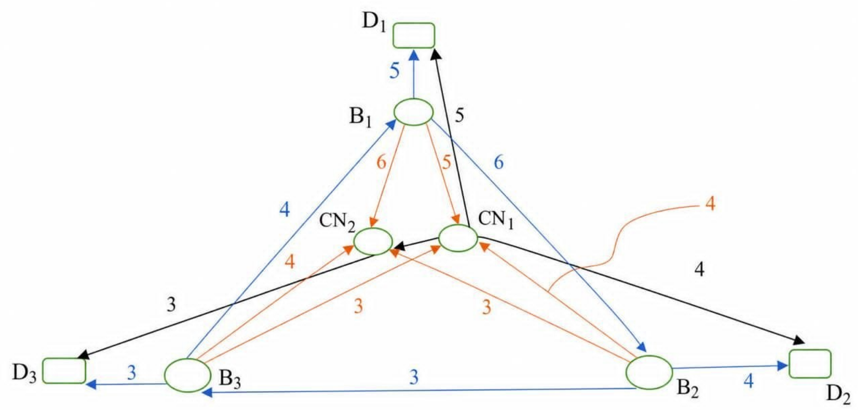


Figure 1: Financial Network

*portion of each bank in the second guarantee fund is 1.) And* $\sum_{i=1}^{3} f_i = 1$ *(The portion of all end-users in the first guarantee fund is 1.),* $\gamma_0 = \frac{\sum_{i=1}^{3} \gamma_i}{3} = \frac{9}{3} = 3$ *(i.e for every central bank* $\gamma_0 = 3$*) So, the totall* $\gamma_0 = 6$*. Thus,* $A_0 = \gamma_0 + \sum_{i=1}^{3} g_i + \sum_{i=1}^{3} f_i + P_i^* = 16$ *(*$P_i^* = 6$ *in the extreme scenario assumed that* $\varphi_i = 0$*).*
*Let* $\sum_{i=1}^{3} Q_i = 4$ *and* $\alpha = 0.25$ *then by the first part of Interbank liability clearing equilibrium,* $Z_i = exp(-0.254) = 0.3679$*. Thus, evaluate the shorfall payable between banks,*

$$S = (6 - 6(0.3679) + (3 - 3(0.3679) + (4 - 4(0.3679)) = 8.2273.$$

*We know that the total payable between banks should be taken is 13. Hence, the amount*

$$\sum_{j=1}^{m} \ell_i \pi_{ji} = 13 - 8.2273 = 4.7727.$$

*Consequently, the total available income is* $A_0 + \sum_{j=1}^{m} \ell_i \pi_{ji} = 16 + 4.7727 = 20.7727$*. Now,*

$$D_i^* = D_i \wedge (A_0 + \sum_{j=1}^{m} \ell_i \pi_{ji}) = 20.7727 \wedge 12 = 12.$$

*And*

$$((A_0 + \sum_{j=1}^{m} \ell_i \pi_{ji}) - D_i)^+ = 20.7727 - 12 = 8.7727 = P_0^*,$$

*it should be distributed equally according to the obligations of banks between each other. i.e,* $\frac{8.7727}{13} \approx 0.67$ *for each share. Every bank returns a loan in the amount listed below.*

$$Bank_1 = 0.67 \times 6 = 4.02,$$
$$Bank_2 = 0.67 \times 4 = 2.68,$$
$$Bank_3 = 0.67 \times 3 = 2.01.$$

*This is the result of having two central banks, however, if there is only one central bank, as proposed in the Amini's paper, it can repay any banks debts as follows.*

$$\begin{gathered}16.7727 - 12 = 4.7727 \textit{ then } \frac{4.7727}{13} \approx 0.37,\\ Bank_1 = 0.37 \times 6 = 2.22,\\ Bank_2 = 0.37 \times 4 = 1.48,\\ Bank_3 = 0.37 \times 3 = 1.11.\end{gathered}$$

## 4. Conclusion

Our findings show that the establishment of two central banks can reduce bank risk, increase aggregate surplus, enhance interbank loan repayment circumstances, and provide citizens better confidence in the protection of their rights.

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