AUT Journal of Mathematics and Computing

AUT J. Math. Comput., 5(4) (2024) 321-327
https://doi.org/10.22060/AJMC.2023.22144.1137

Original Article

Composition operators from Zygmund spaces into Besov Zygmund-type spaces

Hamid Vaezi*[a], Sima Houdfar[a]

[a]*Department of Mathematics, Faculty of Mathematics, Statistics and Computer Sciences, University of Tabriz, Tabriz, Iran*

ABSTRACT: In this paper first, the boundedness and compactness of a composition operator from Zygmund space to Besov Zygmund-type space are studied. Then we study this concepts for this operator by using the hyperbolic-type analytic Besov Zygmund-type class. Finally, we show the relation between the hyperbolic-type analytic Besov Zygmund-type class and the meromorphic (or spherical) Besov Zygmund-type class.

Review History:

Received:30 January 2023
Revised:06 August 2023
Accepted:16 August 2023
Available Online:01 October 2024

Keywords:

Composition operator
Hyperbolic-type analytic Besov Zygmund-type class
Meromorphic (or spherical) Besov Zygmund-type class

MSC (2020):

47B33; 30H40; 30H05

1. Introduction

Let $\mathbb{D}$ be the open unit disc in the complex plane $\mathbb{C}$, $H(\mathbb{D})$ the class of all complex-valued functions analytic on $\mathbb{D}$ and B the family of analytic self-maps φ of the unit disc $\mathbb{D}$. Given an analytic self map φ of $\mathbb{D}$ and an analytic function u on $\mathbb{D}$, the weighted composition operator uC_φ induced by φ and u on $H(\mathbb{D})$ is defined by:

$$uC_\varphi f = u.(f \circ \varphi); f \in H(\mathbb{D}),$$

where the dot denotes pointwise multiplication. The *Bloch space* $\mathcal{B}$, defined as the space of all functions $f \in H(\mathbb{D})$ satisfying

$$\|f\|_{\mathcal{B}} = \sup\{(1-|z|^2)|f'(z)| : z \in \mathbb{D}\} < \infty.$$

$\|.\|_{\mathcal{B}}$ defines a semi-norm on $\mathcal{B}$. We can see that $|f(0)| + \|f\|_{\mathcal{B}}$ is a norm on $\mathcal{B}$ that makes it a Banach space. Any $f \in \mathcal{B}$ satisfies the following growth condition (see, [3]):

$$|f(z)| \leqslant (1 + \log \frac{e}{1-|z|^2})\|f\|_{\mathcal{B}}.$$

*Corresponding author.
E-mail addresses: hvaezi@tabrizu.ac.ir, sima.houdfar@gmail.com

For $1 < p < +\infty$, the Besov space B_p is the space of all analytic functions f on $\mathbb{D}$ satisfying

$$\|f\|_{B_p}^p = \int_{\mathbb{D}} |f'(z)|^p (1-|z|^2)^p d\lambda(z) < \infty,$$

where $d\lambda(z) = (1-|z|^2)^{-2} dA(z)$ is the Mobius invariant measure and dA is the normalized area measure on $\mathbb{D}$. We can see that $|f(0)| + \|f\|_{B_p}$ is a norm on B_p, that makes it a Banach space.
An analytic function f on $\mathbb{D}$ is said to belongs to the *Zygmund space* $\mathcal{Z}$, if

$$\|f\|_{\mathcal{Z}} = \sup_{z\in\mathbb{D}} (1-|z|^2)|f''(z)| < \infty.$$

It is well-known ([7], see Theorem 5.2) that the Zygmund space is contained in the disc algebra. It is easy to check that $\mathcal{Z}$ is a Banach space under the norm $|f(0)| + |f'(0)| + \|f\|_{\mathcal{Z}}$. Also, we can observe that any f belongs to the Zygmund space $\mathcal{Z}$ if and only if f' belongs to the Bloch space $\mathcal{B}$.

For $1 \leqslant p < \infty$, An analytic function f on $\mathbb{D}$ is said to belongs to the *Besov Zygmund-type space* Z_p if

$$\|f\|_{Z_p}^p = \int_{z\in\mathbb{D}} (1-|z|^2)^p |f''(z)|^p d\lambda(z) < \infty.$$

It is easy to check that Z_p is a Banach space under the norm $|f(0)| + |f'(0)| + \|f\|_{Z_p}$. Moreover, we can observe that any f belongs to the Besov Zygmund-type space Z_p if and only if f' belongs to the Besov space B_p.
Since B_p is contained in the Bloch space, it followes that the Z_p is a subset of $\mathcal{Z}$, and hence is contained in disc algebra. For $1 < p < q < \infty$, we have $Z_p \subset Z_q \subset \mathcal{Z}$, and

$$\|f\|_{\mathcal{Z}} \leqslant \|f\|_{Z_q} \leqslant \|f\|_{Z_p}, \quad \text{for any } f \in Z_p.$$

Lemma 1.1. *If $f \in \mathcal{Z}$, then*

I) $|f(z)| \leq \|f\|_{\mathcal{Z}}$ *for all* $z \in \mathbb{D}$,

II) $|f'(z)| \leq \log \dfrac{e}{1-|z|^2} \|f\|_{\mathcal{Z}}$, *for all* $z \in \mathbb{D}$.

Proof. See [12]. □

The following three lemmas are proved in [5].

Lemma 1.2. *For $1 \leqslant p < \infty$ there exists a positive constant C such that if $f \in Z_p$, then*

I) $|f(z) - f(\frac{t}{|z|}z)| \leqslant C\|f\|_{Z_p}(1-|z|)^{\frac{p+1}{2p}}$, *for all* $t \in (0,1)$, $z \in \mathbb{D}\setminus\{0\}$,

II) $|f(z)| \leq \|f\|_{Z_p}$, *for all* $z \in \mathbb{D}$.

Lemma 1.3. *For $1 \leqslant p < \infty$, every sequence in Z_p bounded in norm has a subsequence which converges uniformly in $\overline{\mathbb{D}}$ to a function in Z_p.*

Lemma 1.4. *Let X be a Banach space that is continuously contained in the disc algebra, and let Y be any Banach space of analytic functions on $\mathbb{D}$. Suppose that:*

I) *The point-evaluation functionals on Y are continuous.*

II) *For every sequence $\{f_n\}$ in the unit ball of X there exists $f \in X$ and a subsequence $\{f_{n_j}\}$ such that $f_{n_j} \to f$ uniformly on $\overline{\mathbb{D}}$.*

III) *The operator $T : X \to Y$ is continuous if X has the supremum norm and Y is given the topology of uniform convergence on compact sets.*

Then $T : X \to Y$ is a compact operator if and only if, given a bounded sequence $\{f_n\}$ in X such that $f_n \to 0$ uniformly on $\overline{\mathbb{D}}$, then $\|Tf_n\|_Y \to 0$ as $n \to \infty$.

Remark 1.5. *The proof of the necessity in Lemma 1.4 only uses statements I and III, while the proof of the sufficiency only uses statement II.*

We note that the hypotheses in Lemma 1.4 are satisfies for $X = \mathcal{Z}$ and $Y = Z_p$ by Lemma 1.2 and 1.3. So, the following lemma holds.

Lemma 1.6. *Let $1 \leqslant p < \infty$. If T is a bounded linear operator from $\mathcal{Z}$ into Z_p, then T is compact if and only if $\|Tf_n\|_{Z_p} \to 0$ as $n \to \infty$ for any sequence $\{f_n\}$ in $\mathcal{Z}$ bounded in norm which converges to 0 uniformly on $\overline{\mathbb{D}}$.*

Zygmund-type spaces have attracted a considerable attention recently. Colonna and Li studied the boundedness and compactness of the weighted composition operator from the Bloch space and the analytic Besov spaces into Zygmund space and from H^∞ to Zygmund space in [2, 3], respectively. (See [1, 2, 4, 6, 8, 9, 10, 11, 13] for more results of composition operators, weighted composition operators, and related operators on the Zygmund space and Zygmund-type spaces.)

Boundedness and compactness of weighted composition operators from Z_p into $\mathcal{B}^\alpha$ were studied by Colonna and Tjani in [5]. In this work first, we study the boundedness and compactness of a composition operator from $\mathcal{Z}$ into Z_p, in Section 2. We give Hyperbolic-type analytic Besov Zygmund-type class Z_p^h characterization for boundedness and compactness of a composition operator from $\mathcal{Z}$ into Z_p in Section 3. Finally, in Section 4, we obtain relation between Z_p^h and meromorphic Besov Zygmund-type class $Z_p^{\#}$.

Throughout this paper C denotes a positive constant which may be different at different occurrences.

2. Composition operators from $\mathcal{Z}$ to Z_p

In this section, we study the boundedness and compactness of a composition operator from Zygmund space to Besove Zygmund-type space.

Theorem 2.1. *Let φ be an analytic mapping from $\mathbb{D}$ into itself, $1 \leqslant p < \infty$ and $\varphi \in \mathcal{Z}$. Also, suppose that*

I) $\int_{\mathbb{D}} (\log \frac{e}{1-|\varphi(z)|^2})^p d\lambda(z) < \infty$, *and*

II) $\int_{\mathbb{D}} (\frac{1-|z|^2}{1-|\varphi(z)|^2})^p (\log \frac{e}{1-|z|^2})^{2p} d\lambda(z) < \infty$.

Then the composition operator $C_\varphi : \mathcal{Z} \to Z_p$ is compact.

Proof. Let $\{g_k\}$ be a sequence in $\mathcal{Z}$ bounded in norm which converges to zero uniformly in $\overline{\mathbb{D}}$. Using conditions I, II and Lemma 1.1, there exists a constant C (which is different from one occurrence to the other) such that

$$\begin{aligned}
\|C_\varphi g_k\|_{Z_p} &= \left(\int_{\mathbb{D}} (1-|z|^2)^p |(g_k(\varphi(z)))''|^p d\lambda(z) \right)^{\frac{1}{p}} \\
&= \left(\int_{\mathbb{D}} (1-|z|^2)^p |\varphi''(z)|^p |g_k'(\varphi(z))|^p d\lambda(z) + \int_{\mathbb{D}} (1-|z|^2)^p |\varphi'(z)|^p |\varphi'(z)|^p |g_k''(\varphi(z))|^p d\lambda(z) \right)^{\frac{1}{p}} \\
&\leqslant C \left(\|\varphi\|_{\mathcal{Z}}^p \int_{\mathbb{D}} |g_k'(\varphi(z))|^p d\lambda(z) \right)^{\frac{1}{p}} + C \left(\|g_k\|_{\mathcal{Z}}^p \int_{\mathbb{D}} (\frac{1-|z|^2}{1-|\varphi(z)|^2})^p |\varphi'(z)|^p |\varphi'(z)|^p d\lambda(z) \right)^{\frac{1}{p}} \\
&\leqslant C \|\varphi\|_{\mathcal{Z}} \|g_k\|_{\mathcal{Z}} \left(\int_{\mathbb{D}} (\log \frac{e}{1-|\varphi(z)|^2})^p d\lambda(z) \right)^{\frac{1}{p}} \\
&\quad + C \|g_k\|_{\mathcal{Z}} \|\varphi\|_{\mathcal{Z}}^2 \left(\int_{\mathbb{D}} (\frac{1-|z|^2}{1-|\varphi(z)|^2})^p (\log \frac{e}{1-|z|^2})^{2p} d\lambda(z) \right)^{\frac{1}{p}} \longrightarrow 0.
\end{aligned}$$

Thus, by Lemma 1.6, the operator $C_\varphi : \mathcal{Z} \to Z_p$ is compact. □

Note. It follows that if the conditions (1) and (2) of above theorem hold, the composition operator $C_\varphi : \mathcal{Z} \to Z_p$ is bounded. In the following theorem, we prove this result straightforward.

Theorem 2.2. *Let φ be an analytic mapping from $\mathbb{D}$ into itself, $1 \leqslant p < \infty$ and $\varphi \in \mathcal{Z}$. Also, suppose that*

I) $\int_{\mathbb{D}} (\log \frac{e}{1-|\varphi(z)|^2})^p d\lambda(z) < \infty$, *and*

II) $\int_{\mathbb{D}} (\frac{1-|z|^2}{1-|\varphi(z)|^2})^p (\log \frac{e}{1-|z|^2})^{2p} d\lambda(z) < \infty$.

Then the composition operator $C_\varphi : \mathcal{Z} \to Z_p$ is bounded.

Proof. Suppose $f \in \mathcal{Z}$. The proof of the Theorem 2.1 shows that by conditions I, II and Lemma 1.1, there exists a constant C (which is different from one occurrence to the other) such that

$$\begin{aligned}\|C_\varphi f\|_{Z_p} &= \left(\int_{\mathbb{D}} (1-|z|^2)^p |(f(\varphi(z)))''|^p d\lambda(z)\right)^{\frac{1}{p}} \\ &\leqslant C\|\varphi\|_{\mathcal{Z}}\|f\|_{\mathcal{Z}} \left(\int_{\mathbb{D}} (\log \frac{e}{1-|\varphi(z)|^2})^p d\lambda(z)\right)^{\frac{1}{p}} + C\|f\|_{\mathcal{Z}}\|\varphi\|^2_{\mathcal{Z}} \left(\int_{\mathbb{D}} (\frac{1-|z|^2}{1-|\varphi(z)|^2})^p (\log \frac{e}{1-|z|^2})^{2p} d\lambda(z)\right)^{\frac{1}{p}} \\ &\leqslant C\|f\|_{\mathcal{Z}}.\end{aligned}$$

Thus, $C_\varphi : \mathcal{Z} \to Z_p$ is bounded. □

3. Hyperbolic-type analytic Besov Zygmund-type class

Take $\varphi \in B$ such that $|\varphi'(z)| < 1$. By the Schwartz-Pick lemma $\sup_{z\in\mathbb{D}}(1-|z|^2)\varphi^{**}(z) \leq 1$, where $\varphi^{**}(z)$ is the hyperbolic-type derivative

$$\varphi^{**}(z) = \frac{|\varphi''(z)|}{1-|\varphi'(z)|^2}.$$

Definition 3.1. *For $1 \leqslant p < +\infty$, the hyperbolic-type analytic Besov Zygmund-type class Z_p^h is defined to be the family of all functions $\varphi \in B$ whit $|\varphi'(z)| < 1$ such that*

$$\|\varphi\|^p_{Z_p^h} = \int_{\mathbb{D}} ((1-|z|^2)\varphi^{**}(z))^p d\lambda(z) < \infty.$$

In this section by using the hyperbolic-type analytic Besov Zygmund-type class Z_p^h, we characterize the compactness and boundedness of the composition operator $C_\varphi : \mathcal{Z} \to Z_p$.

Theorem 3.2. *Let φ be an analytic mapping from $\mathbb{D}$ into itself, $1 \leqslant p < \infty$ and $\varphi \in Z_p^h$. Also, suppose that*

I) $\log \dfrac{e}{1-|\varphi(z)|^2}$ *is bounded on $\mathbb{D}$, and*

II) $\displaystyle\int_{\mathbb{D}} \frac{(1-|z|^2)^p}{(1-|\varphi(z)|^2)^p} d\lambda(z) < \infty.$

Then the composition operator $C_\varphi : \mathcal{Z} \to Z_p$ is compact.

Proof. Let $\{g_k\}$ be a sequence in $\mathcal{Z}$ bounded in norm which converges to zero uniformly in $\overline{\mathbb{D}}$. By using conditions I, II and lemma 1.1, there exists a constant C such that

$$\begin{aligned}\|C_\varphi(g_k)\|_{Z_p} &= \left(\int_{\mathbb{D}} |[g_k(\varphi(z))]''|^p (1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}} \\ &= \left(\int_{\mathbb{D}} |\varphi''(z)|^p |g_k'(\varphi(z))|^p (1-|z|^2)^p d\lambda(z) \right.\\ &\quad \left. + \int_{\mathbb{D}} |\varphi'(z)|^p |\varphi'(z)|^p |g_k''(\varphi(z))|^p (1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}} \\ &\leqslant C(1-|\varphi'(z)|^2)^p \left(\int_{\mathbb{D}} (\varphi^{**}(z))^p |g_k'(\varphi(z))|^p (1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}} \\ &\quad + C|\varphi'(z)|^{2p} \left(\int_{\mathbb{D}} |g_k''(\varphi(z))|^p (1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}}\end{aligned}$$

$$\leqslant C\left(\int_{\mathbb{D}}(\log\frac{e}{1-|\varphi(z)|^2})^p\|g_k\|^p_{\mathcal{Z}}(\varphi^{**}(z))^p(1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}}$$

$$+C\|g_k\|_{\mathcal{Z}}\left(\int_{\mathbb{D}}\frac{(1-|z|^2)^p}{(1-|\varphi(z)|^2)^p}d\lambda(z)\right)^{\frac{1}{p}}$$

$$\leqslant C\|g_k\|_{\mathcal{Z}}\left(\int_{\mathbb{D}}(\varphi^{**}(z))^p(1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}}+C\|g_k\|_{\mathcal{Z}}$$

$$\leqslant C\|g_k\|_{\mathcal{Z}}\|\varphi\|_{Z_p^h}+C\|g_k\|_{\mathcal{Z}}\longrightarrow 0.$$

From Lemma 1.6, it follows that the operator $C_\varphi : \mathcal{Z} \to Z_p$ is compact. □

Note. It follows that if the conditions I and II of above theorem hold, the composition operator $C_\varphi : \mathcal{Z} \to Z_p$ is bounded. In the following theorem, we prove this result straightforward.

Theorem 3.3. *Let φ be an analytic mapping from $\mathbb{D}$ into itself, $1 \leqslant p < \infty$ and $\varphi \in Z_p^h$. Also, suppose that*

I) $\log\dfrac{e}{1-|\varphi(z)|^2}$ *is bounded on $\mathbb{D}$, and*

II) $\displaystyle\int_{\mathbb{D}}\frac{(1-|z|^2)^p}{(1-|\varphi(z)|^2)^p}d\lambda(z) < \infty.$

Then the composition operator $C_\varphi : \mathcal{Z} \to Z_p$ is bounded.

Proof. The proof of the Theorem 3.2 shows that for any $f \in \mathcal{Z}$, there exists a constant C such that

$$\|C_\varphi(f)\|_{Z_p}=\left(\int_D|[f(\varphi(z))]''|^p(1-|z|^2)^p d\lambda(z)\right)^{\frac{1}{p}}$$
$$\leqslant C\|f\|_{\mathcal{Z}}\|\varphi\|_{Z_p^h}+C\|f\|_{\mathcal{Z}}\leqslant C\|f\|_{\mathcal{Z}}.$$

Thus, $C_\varphi : \mathcal{Z} \to Z_p$ is bounded. □

4. Relation between $Z_p^\#$ and Z_p^h

Since analytic functions f in the unit disc $\mathbb{D}$ are also meromorphic in $\mathbb{D}$, we can study the class of meromorphic functions, provided that the ordinary derivative of f is replaced by the spherical-type derivative $f^{\#\#}$, where

$$f^{\#\#}(z)=\frac{|f''(z)|}{1+|f'(z)|^2},\qquad (z\in\mathbb{D}).$$

The family of *normal meromorphic-type* functions in $\mathbb{D}$ is denoted by $\mathcal{N}$ and is defined by

$$\mathcal{N}=\{f \text{ meromorphic in } \mathbb{D} : \sup_{z\in\mathbb{D}}(1-|z|^2)f^{\#\#}(z)<\infty\}.$$

We define

$$\|f\|_{\mathcal{N}}=\sup_{z\in\mathbb{D}}(1-|z|^2)f^{\#\#}(z).$$

Definition 4.1. *For $1 \leqslant p < \infty$, we define the meromorphic (or spherical-type) Besov Zygmund-type class, $Z_p^\#$ by*

$$Z_p^\#=\{f \textit{ meromorphic in } \mathbb{D} : \int_{\mathbb{D}}(f^{\#\#}(z))^p(1-|z|^2)^{p-2}dA(z)<\infty\}.$$

We define

$$\|f\|^p_{Z_p^\#}=\int_{\mathbb{D}}(f^{\#\#}(z))^p(1-|z|^2)^{p-2}dA(z).$$

The following theorem shows a relation between the hyperbolic-type analytic Besov Zygmund-type class and the meromophic (or spherical) Besov Zygmund-type class.

Proposition 4.2. *Let $1 \le p < \infty$, f is a normal meromorphic function in $\mathbb{D}$ and $\varphi \in Z_p^h$. Also, suppose that*

I) $\dfrac{|f'(\varphi(z))|^p}{(1+|\varphi'(z)f'(\varphi(z))|^2)^p}$ *is bounded on* $\mathbb{D}$, *and*

II) $\displaystyle\int_{\mathbb{D}}(1-|z|^2)^{p-2}\frac{(1+|f'(\varphi(z))|^2)^p}{(1-|\varphi'(z)f'(\varphi(z))|^2)^p(1-|\varphi(z)|^2)^p}dA(z)<\infty.$

Then $f\circ\varphi\in Z_p^{\#}$.

Proof. Since $\varphi\in Z_p^h$, we have

$$\|\varphi\|^p_{Z_p^h}=\int_{\mathbb{D}}(1-|z|^2)^{p-2}(\varphi^{**}(z))^p dA(z)<\infty.$$

So, there exists a constant C such that

$$\begin{aligned}\|f\circ\varphi\|^p_{Z_p^{\#}}&=\int_{\mathbb{D}}(1-|z|^2)^{p-2}((f\circ\varphi)^{\#\#}(z))^p dA(z)\\&\leqslant C\int_{\mathbb{D}}(1-|z|^2)^{p-2}\frac{|\varphi''(z)|^p|f'(\varphi(z))|^p+|\varphi'(z)|^p|\varphi'(z)|^p|f''(\varphi(z))|^p}{(1+|\varphi'(z)f'(\varphi(z))|^2)^p}dA(z)\\&\leqslant C(1-|\varphi'(z)|^2)^p\int_{\mathbb{D}}(1-|z|^2)^{p-2}\frac{|\varphi''(z)|^p}{(1-|\varphi'(z)|^2)^p}\frac{|f'(\varphi(z))|^p}{(1+|\varphi'(z)f'(\varphi(z))|^2)^p}dA(z)\\&\quad+C|\varphi'(z)|^{2p}\int_{\mathbb{D}}(1-|z|^2)^{p-2}\frac{|f''(\varphi(z))|^p}{(1+|\varphi'(z)f'(\varphi(z))|^2)^p}(\frac{1+|f'(\varphi(z)|^2}{1+|f'(\varphi(z)|^2})^p(\frac{1-|\varphi(z)|^2}{1-|\varphi(z)|^2})^p dA(z)\\&\leqslant C\|\varphi\|^p_{Z_p^h}+\|f\|^p_{\mathcal{N}}\int_{\mathbb{D}}(1-|z|^2)^{p-2}\frac{(1+|f'(\varphi(z))|^2)^p}{(1+|\varphi'(z)f'(\varphi(z))|^2)^p(1-|\varphi(z)|^2)^p}dA(z)<\infty.\end{aligned}$$

Thus, $f\circ\varphi\in Z_p^{\#}$. □

References

[1] E. Abbasi and H. Vaezi, *Generalized weighted composition operators from the Bloch-type spaces to the weighted Zygmund spaces*, Filomat, 33 (2019), pp. 981–992.

[2] F. Colonna and S. Li, *Weighted composition operators from H^∞ into the Zygmund spaces*, Complex Anal. Oper. Theory, 7 (2013), pp. 1495–1512.

[3] ——, *Weighted composition operators from the bloch space and the analytic besov spaces into the zygmund space*, Journal of Operators, 2013 (2013), p. 154029.

[4] ——, *Weighted composition operators from the Lipschitz space into the Zygmund space*, Math. Inequal. Appl., 17 (2014), pp. 963–975.

[5] F. Colonna and M. Tjani, *Weighted composition operators from the Besov spaces into the weighted-type space H^∞_μ*, J. Math. Anal. Appl., 402 (2013), pp. 594–611.

[6] J. Du, S. Li, and Y. Zhang, *Essential norm of generalized composition operators on Zygmund type spaces and Bloch type spaces*, Ann. Polon. Math., 119 (2017), pp. 107–119.

[7] P. L. Duren, *Theory of H^p spaces*, Pure and Applied Mathematics, Vol. 38, Academic Press, New York-London, 1970.

[8] Q. Hu, S. Li, and Y. Zhang, *Essential norm of weighted composition operators from analytic Besov spaces into Zygmund type spaces*, J. Contemp. Mathemat. Anal., 54 (2019), pp. 129–142.

[9] H. Li and Z. Guo, *On a product-type operator from Zygmund-type spaces to Bloch-Orlicz spaces*, J. Inequal. Appl., (2015), pp. 2015:132, 18.

[10] A. H. Sanatpour and M. Hassanlou, *Essential norms of weighted differentiation composition operators between Zygmund type spaces and Bloch type spaces*, Filomat, 31 (2017), pp. 2877–2889.

[11] S. Stević, *Composition operators from the Hardy space to Zygmund-type spaces on the upper half-plane and the unit disc*, J. Comput. Anal. Appl., 12 (2010), pp. 305–312.

[12] S. Ye and Q. Hu, *Weighted composition operators on the Zygmund space*, Abstr. Appl. Anal., (2012), pp. Art. ID 462482, 18.

[13] X. Zhu, *Integral-type operators from iterated logarithmic Bloch spaces to Zygmund-type spaces*, Appl. Math. Comput., 215 (2009), pp. 1170–1175.

Please cite this article using:

Hamid Vaezi, Sima Houdfar, Composition operators from Zygmund spaces into Besov Zygmund-type spaces, AUT J. Math. Comput., 5(4) (2024) 321-327
https://doi.org/10.22060/AJMC.2023.22144.1137