Original Article

# On a group of the form $2^{11}{:}M_{24}$

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**ABSTRACT:** The Conway group $Co_1$ is one of the 26 sporadic simple groups. It is the largest of the three Conway groups with order $4157776806543360000 = 2^{21}.3^9.5^4.7^2.11.13.23$ and has 22 conjugacy classes of maximal subgroups. In this paper, we discuss a group of the form $\overline{G} = N{:}G$, where $N = 2^{11}$ and $G = M_{24}$. This group $\overline{G} = N{:}G = 2^{11}{:}M_{24}$ is a split extension of an elementary abelian group $N = 2^{11}$ by a Mathieu group $G = M_{24}$. Using the computed Fischer matrices for each class representative $g$ of $G$ and ordinary character tables of the inertia factor groups of $G$, we obtain the full character table of $\overline{G}$. The complete fusion of $\overline{G}$ into its mother group $Co_1$ is also determined using the permutation character of $Co_1$.

## 1. Introduction

The group $\overline{G}$ has order 501397585920 and $[Co_1 : \overline{G}] = 8292375$. In this paper, we will compute the conjugacy classes, inertia factor groups and Fischer-Cliffford matrices which lead to the construction of the ordinary character table of $\overline{G} = N{:}G = 2^{11}{:}M_{24}$ using the technique of coset analysis and the theory of Fischer-Clifford matrices. These methods have been largely studied and applied by many researchers in their journals, PhD and Masters Dissertations (see for example: [1, 2, 3, 4, 5, 7, 9, 10, 13, 14]).

The fusion of the classes of $\overline{G}$ into the classes of its mother group $Co_1$ will be completely obtained by the permutation character of $Co_1$ on $\overline{G}$.

To construct $\overline{G}$ inside its cover $Co_1$, we first need to construct $Co_1$ as follows:
From the online Atlas $V_3$ [19], we obtain two $24 \times 24$ matrix generators $a$ and $b$ of $Co_1$ over GF(2) with $o(a) = 2$ and $o(b) = 3$. Then with the help of GAP [12], $Co_1 = \langle\{a, b\}\rangle$. Again from online Atlas $V_3$ [19], We then extract **programme A** (see Section 7) and implement it in GAP [12] to obtain two $24 \times 24$ matrix generators $c$ and $d$ of

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$\overline{G}$ over GF(2) with $o(c) = 2$ and $o(d) = 6$ presented below.

$$
c = \begin{pmatrix} \text{(24×24 binary matrix)} \end{pmatrix} \qquad d = \begin{pmatrix} \text{(24×24 binary matrix)} \end{pmatrix}.
$$

Thus, we have that $\overline{G} = 2^{11}{:}M_{24} = \langle c, d \rangle$. After constructing $\overline{G}$ in GAP, we observe that it has two non-trivial normal subgroups and only one of these is of order 2048 and is an elementary abelian 2-group with 11 generators $n_1$, $n_2, n_3$, $n_4$, $n_5$, $n_6$, $n_7$, $n_8$, $n_9$, $n_{10}$ and $n_{11}$ each of order 2. This group is our $\{N\}$ and $\{N\} = 2^{11}$. Therefore, up to isomorphism, there is only one group of the type $2^{11}{:}M_{24}$ inside $Co_1$.

The complement of $N = 2^{11}$ in $\overline{G} = 2^{11}{:}M_{24}$ is $G = M_{24}$ generated by two $24 \times 24$ matrices each of order 2 which are then reduced to two $11 \times 11$ matrices $g_1$ and $g_2$ using **Programmme F** (see section 7).

$$
g_1 = \begin{pmatrix} \text{(11×11 binary matrix)} \end{pmatrix} \qquad g_2 = \begin{pmatrix} \text{(11×11 binary matrix)} \end{pmatrix} \quad \text{with } o(g_1) = 2 \text{ and } o(g_2) = 3
$$

## 2. The Action of $G = M_{24}$ on $2^{11}$ and $Irr(2^{11})$

$\bar{G} = 2^{11}{:}M_{24}$ is a split extension of $N = 2^{11}$ by $G = M_{24}$, where $N$ is considered as vector space of dimension 11 over $GF(2)$. Using GAP, the action of $G = M_{24}$ on $N = 2^{11}$ results into 3 orbits of lengths 1, 759 and 1288 with corresponding point stabilizers $M_{24}$, $2^4{:}A_8$ and $M_{12}{:}2$ of orders 244823040, 322560 and 190080 respectively. Since $G$ has 3 orbits from its action on $N$, it must have 3 orbits from its action on $Irr(2^{11})$ by Brauer's theorem in [18, Theorem 2.6.7]. This is exactly the case here, we have 3 orbits from the action on the conjugacy classes and the irreducible characters. That is, we have 1, 759 and 1288 orbit lengths from the action on the conjugacy classes and 1, 276 and 1771 on the irreducible characters.

## 3. Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$

In this section, the technique of coset analysis is used to compute the conjugacy classes of the elements of $\overline{G}$. This technique was developed and first used by Moori [18] and since then, it has been applied by many other researchers to calculate the conjugacy classes of groups of extension type. We give a brief summary of coset analysis. We look at the action of $\bar{G}$ on $N\bar{g}$, for the split extension it suffices to look at $Ng$, $g \in G$. First $N$ acts on $Ng$ and breaks it into k orbits say $Q_1, Q_2, \ldots, Q_k$. Then we act $C_G(g)$ on these orbits and $f_j$ of them fuse to form one orbit $\triangle_j$ with $\sum_j f_j = k$. For this we use **Programme B** and **Programme C** for the power maps in [9]. The reader is referred to Basheer [3], Chikopela [6], Chileshe [9] and Kapata [13] among others for further details and recent application of this technique. Now, corresponding to 26 conjugacy classes of $M_{24}$, we obtain 80 conjugacy classes of $\overline{G}$ which are listed in Table 1. In this table, the $m_j's$ are the weights which will play a big role in computation of Clifford-Fischer matrices later in section 3 and are calculated by $m_j = \frac{f_j \times |N|}{k}$.

**Theorem 3.1.** *Let $G$ be a group, $p$ be the number of conjugacy classes of $G$ and $Irr(G) = \{\chi_1, \chi_2, \cdots, \chi_q\}$ be the set of irreducible characters of $G$, then $p = q$.*

**Proof.** (See Chileshe [8]). $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

## 4. Fischer-Clifford Matrices of $\bar{G}$

After obtaining the conjugacy classes of $\overline{G}$ and the fusion maps of the inertia factor groups $H_2$ and $H_3$ into $G$, the Fischer-Clifford matrices of the group $\bar{G} = 2^{11}{:}M_{24}$ will now be computed. **Programme D** is extensively used for the automatic determination of a possible candidate for each Fischer-Clifford matrix $M(g)$, $g \in G$, of $\overline{G}$. The properties of Fischer-Clifford matrices discussed in [3], [10], [16], and [17] are used to rearrange the rows and columns in order to get the unique matrix $M(g)$ for $\bar{G} = 2^{11}{:}M_{24}$. Note that **Programme D** only applies on split extensions $N{:}G$, where $N$ is elementary abelian. Since $N = 2^{11}$ is an elementary abelian $p$-group, then all the following properties of the Fischer matrix $M(g)$ that help with the computation of its entries will hold:

(1) $a_j^{(1,g)} = 1$ for all $j \in \{1, 2, \ldots, c(g)\}$,

(2) $|X(g)| = |R(g)|$,

(3) $\displaystyle\sum_{(i,y) \in R(g)} \frac{|C_{H_i}(y)|a_{(i,y)}^j \overline{a_{(i,y)}^{j'}}}{|C_{\bar{G}}(x_j)|} = \delta_{jj'}$, called the column orthogonality relation,

(4) $a_1^{(i,y)} = \dfrac{|C_G(g)|}{|C_{H_i}(y)|}$,

(5) $|a_1^{(i,y)}| \geq |a_j^{(i,y)}|$.

For example, let us consider the conjugacy class $3A$ of $G = M_{24}$, then applying **property (e)** of **Theorem 5.2.4** in [15], $M(3A)$ takes the following form with corresponding weights attached to the rows and columns,

$$
\begin{array}{cc}
 & \begin{array}{cccc} |C_{\overline{G}}(3A)| & |C_{\overline{G}}(6A)| & |C_{\overline{G}}(6B)| & |C_{\overline{G}}(6C)| \\ 34560 & 2304 & 3456 & 5760 \end{array} \\
\begin{array}{l} |C_{H_1}(3A)| = \quad 1080 \\ |C_{H_2}(3a)| = \quad 72 \\ |C_{H_3}(3a)| = \quad 1080 \\ |C_{H_3}(3b)| = \quad 72 \\ \phantom{|C_{H_3}(3b)| =} m_{ij} \end{array} &
\left(\begin{array}{cccc}
1 & 1 & 1 & 1 \\
15 & a & b & c \\
1 & d & e & f \\
15 & g & h & i \\
64 & 960 & 640 & 384
\end{array}\right)
\end{array}
$$

In order to determine the entries $a$, $b$, $c$, $d$, $e$, $f$, $g$, $h$ and $i$ of the Fischer-Clifford matrix $M(3A)$, the GAP output for programme D is first generated and arranged using properties of Fischer-Clifford matrices to obtain:

$$
M(3A) = \begin{pmatrix}
1 & 1 & 1 & 1 \\
15 & -1 & 3 & -5 \\
1 & 1 & -1 & -1 \\
15 & -1 & -3 & 5
\end{pmatrix}
$$

Similarly,

$$
M(1A) = \begin{pmatrix}
1 & 1 & 1 \\
276 & 20 & -12 \\
1771 & -21 & 11
\end{pmatrix}
$$

$$M(21A) = (1)$$

$$M(21B) = (1)$$

$$M(23A) = (1)$$

$$M(23B) = (1)$$

Proceeding this way on every conjugacy class of G, we obtain all the 26 corresponding Fischer-Clifford matrices of $\bar{G}$. **Programme G** computes the Fischer Matrix of $\bar{G}$ on $1_G \in G$. We list all the Fischer matrices in Table 7.

## 5. Character Table of $\bar{G}$

By Theorem 3.1, the character table is easily seen as a square matrix with columns indexed by the conjugacy classes and rows corresponding to the irreducible characters of $G$.

**Definition 5.1** ([9]). *Let $G$ be a group and $\{g_1, g_2, \cdots, g_l\}$ be the conjugacy class representatives of $G$. Let $Irr(G) = \{\chi_1, \chi_2, \cdots, \chi_l\}$ be a set of irreducible characters of $G$. Then the **character table** of $G$ is an $l \times l$ matrix whose entries are the values $\chi_i(g_j)$ for $i, j \in \{1, 2, \cdots, l\}$. The columns of this matrix are indexed by $g_j$ and the rows by $\chi_i$ for $i, j \in \{1, 2, \cdots, l\}$.*

Now, once we have obtained the conjugacy classes of $\bar{G} = 2^{11}{:}M_{24}$, the character tables of all the inertia factor groups $H_1$, $H_2$ and $H_3$ presented in Tables 4, 5 and 6 respectively, the fusions of conjugacy classes of the inertia factor groups $H_1$ and $H_2$ into classes of $G = M_{24}$ given in Tables 2 and 3 respectively and the Fischer-Clifford matrices of $\bar{G} = 2^{11}{:}M_{24}$ given in Table 7, the full character table of $\bar{G}$ can then be constructed. The character table of $\bar{G} = 2^{11}{:}M_{24}$ is partitioned row-wise into 3 blocks, where each block corresponds to an inertia factor group $H_i$. Given the character table of the inertia factor group $H_i$, we take the columns of this character table and multiply them by the corresponding rows of the Fischer-Clifford matrix denoted by $M(g)$ and then we fill the portions of the character table of $\bar{G}$ for its classes which come from the coset $Ng$. That is, we multiply the appropriate partial character tables of the inertia factor groups by the corresponding rows of the Fischer matrices to obtain the full character table of $\bar{G}$. Again, by Theorem 3.1, since $\bar{G}$ has 80 conjugacy classes, it must have 80 irreducible characters. For instance, on the identity of $G$ we have

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 276 & 20 & -12 \\ 1771 & -21 & 11 \end{pmatrix}.$$

Multiplying each row of M(1A) by 1st columns of the corresponding Tables 4, 5 and 6 (Character tables of inertia factor groups; $H_1$, $H_2$ and $H_3$), we obtain the values of the characters of $\bar{G}$ on $\bar{G}$-classes $1A$, $2A$ and $2B$ and able to fill the 1st portions of the 1st, 2nd and third blocks of the character table of $\bar{G}$ as follows:

1. If column 1 of Table 1.3 is multiplied by row 1 of M(1A), we get:

$$\begin{pmatrix} 1 \\ 23 \\ 45 \\ 45 \\ 231 \\ 231 \\ 252 \\ 253 \\ 483 \\ 770 \\ 770 \\ 990 \\ 990 \\ 1035 \\ 1035 \\ 1035 \\ 1265 \\ 1771 \\ 2024 \\ 2277 \\ 3312 \\ 3520 \\ 5313 \\ 5544 \\ 5796 \\ 10395 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 23 & 23 & 23 \\ 45 & 45 & 45 \\ 45 & 45 & 45 \\ 231 & 231 & 231 \\ 231 & 231 & 231 \\ 252 & 252 & 252 \\ 253 & 253 & 253 \\ 483 & 483 & 483 \\ 770 & 770 & 770 \\ 770 & 770 & 770 \\ 990 & 990 & 990 \\ 990 & 990 & 990 \\ 1035 & 1035 & 1035 \\ 1035 & 1035 & 1035 \\ 1035 & 1035 & 1035 \\ 1265 & 1265 & 1265 \\ 1771 & 1771 & 1771 \\ 2024 & 2024 & 2024 \\ 2277 & 2277 & 2277 \\ 3312 & 3312 & 3312 \\ 3520 & 3520 & 3520 \\ 5313 & 5313 & 5313 \\ 5544 & 5544 & 5544 \\ 5796 & 5796 & 5796 \\ 10395 & 10395 & 10395 \end{pmatrix}$$

2. If column 1 of Table 1.4 is multiplied by row 2 of M(1A), we get

$$
\begin{pmatrix} 1 \\ 1 \\ 21 \\ 21 \\ 45 \\ 45 \\ 45 \\ 45 \\ 55 \\ 99 \\ 99 \\ 154 \\ 154 \\ 210 \\ 210 \\ 231 \\ 231 \\ 231 \\ 385 \\ 385 \\ 560 \end{pmatrix}
\times \begin{pmatrix} 276 & 20 & -12 \end{pmatrix} =
\begin{pmatrix}
276 & 20 & -12 \\
276 & 20 & -12 \\
5796 & 420 & -252 \\
5796 & 420 & -252 \\
12420 & 900 & -540 \\
12420 & 900 & -540 \\
12420 & 9002 & -540 \\
12420 & 900 & -540 \\
15180 & 1100 & -660 \\
15180 & 1100 & -660 \\
27324 & 1980 & -1188 \\
27324 & 1980 & -1188 \\
42504 & 3080 & -1848 \\
42504 & 3080 & -1848 \\
57960 & 4200 & -2520 \\
57960 & 4200 & -2520 \\
63756 & 4620 & -2772 \\
63756 & 4620 & -2772 \\
154560 & 11200 & -6720 \\
106260 & 7700 & -4620 \\
106260 & 7700 & -4620
\end{pmatrix}
$$

3. If column 1 of Table 1.5 is multiplied by row 3 of M(1A), we have that

$$
\begin{pmatrix} 1 \\ 1 \\ 5 \\ 5 \\ 5 \\ 5 \\ 6 \\ 6 \\ 9 \\ 9 \\ 10 \\ 10 \\ 12 \\ 16 \\ 18 \\ 18 \\ 18 \\ 30 \\ 45 \\ 45 \\ 45 \\ 45 \\ 72 \\ 72 \\ 90 \\ 90 \\ 90 \\ 90 \\ 108 \\ 135 \\ 135 \\ 135 \\ 135 \end{pmatrix}
\times \begin{pmatrix} 1771 & -21 & 11 \end{pmatrix} =
\begin{pmatrix}
1771 & -21 & 11 \\
1771 & -21 & 11 \\
8855 & -105 & 55 \\
8855 & -105 & 55 \\
8855 & -105 & 55 \\
8855 & -105 & 55 \\
15939 & -189 & 99 \\
15939 & -189 & 99 \\
17710 & -210 & 110 \\
17710 & -210 & 110 \\
28336 & -336 & 176 \\
10626 & -126 & 66 \\
10626 & -126 & 66 \\
21252 & -252 & 132 \\
31878 & -378 & 198 \\
53130 & -630 & 330 \\
31878 & -378 & 198 \\
31878 & -378 & 198 \\
79695 & -945 & 495 \\
79695 & -945 & 495 \\
79695 & -945 & 495 \\
79695 & -945 & 495 \\
127512 & -1512 & 792 \\
127512 & -1512 & 792 \\
159390 & -1890 & 990 \\
159390 & -1890 & 990 \\
159390 & -1890 & 990 \\
159390 & -1890 & 990 \\
191268 & -2268 & 118 \\
239085 & -2835 & 1485 \\
239085 & -2835 & 1485 \\
239085 & -2835 & 1485 \\
239085 & -2835 & 1485
\end{pmatrix}
$$

Carrying out this operation on all other classes of G, we are able to obtain all the 80 irreducible characters of $\bar{G}$ given in Table 8. The Table is divided into three blocks corresponding to three inertia groups $\bar{G}=\bar{H}_1$, $\bar{H}_2$ and $\bar{H}_3$. The consistency of the Character Table was checked using **Programme E**.

## 6. Fusion of $\bar{G} = 2^{11}{:}M_{24}$ into $Co_1$

We first state and prove the following theorem.

**Theorem 6.1 ([15]).** *Let $\phi$ be a character of H and let $\phi \uparrow_H^G$ be the induced character from H to G. Let $g \in G$ and suppose that $[g]$ breaks into m classes in H with representatives $x_1, x_2, \ldots, x_m$. If $H \cap [g] = \emptyset$, then $\phi \uparrow_H^G (g) = 0$, while if $H \cap [g] \neq \emptyset$, then*

$$\phi \uparrow_H^G (g) = |C_G(g)| \sum_{i=1}^m \frac{\phi(x_i)}{|C_H(x_i)|}.$$

**Proof.** We have

$$\phi \uparrow_H^G (g) = \frac{1}{|H|} \sum_{x \in G} \phi^0(xgx^{-1}).$$

If $H \cap [g] = \emptyset$, then $xgx^{-1} \notin H, \forall x \in G$ and thus $\phi^0(xgx^{-1}) = 0$ and we have that $\phi \uparrow_H^G (g) = 0$.
Now if $H \cap [g] \neq \emptyset$, then let $h \in H \cap [g]$. As $x$ runs over G, we have $xgx^{-1} = h$ for exactly $|C_G(g)|$ times, so $\phi \uparrow_H^G (g) = \frac{|C_G(g)|}{|H|} \sum_{y \in [g]} \phi^0(y)$. Now $\phi^0(y) = 0$ if $y \notin H$ and $[g] \cap H$ contains $[H : C_H(x_i)]$ conjugates of each $x_i$.
Therefore

$$\phi \uparrow_H^G (g) = |C_G(g)| \sum_{i=1}^m \frac{\phi(x_i)}{|C_H(x_i)|}.$$

$\square$

**Corollary 6.2.** *Let $H \leq G$. Let $g \in G$ and $x_1, x_2, \ldots, x_m$ be representatives of the conjugacy classes of H that fuse to $[g]$. Then*

$$\chi(G/H)(g) = \sum_{i=1}^m \frac{|C_G(g)|}{|C_H(x_i)|}.$$

**Proof.** This follows from Theorem 5.1 $\square$

We now use the results of Theorem 6.1 and Corollary 6.2 to fuse the classes of $\bar{G}$ into the classes of $Co_1$. Let $\chi(Co_1|\bar{G})$ denote the permutation character of $CO_1$ on the cosets of $\bar{G}$ in $Co_1$. Let $[g]_{Co_1}$ denote a conjugacy class of $Co_1$ with representative $g$ and similarly $[\bar{g}]_{\bar{G}}$ denote a conjugacy class of $\bar{G}$ with representative $\bar{g}$. We recall that

$$\begin{cases} \text{if } \bar{G} \cap [g]_{Co_1} = \emptyset, \text{ then } \chi(Co_1|\bar{G})(g) = 0 \\[3mm] \text{if } \bar{G} \cap [g]_{Co_1} \neq \emptyset, \text{ then } \chi(Co_1|\bar{G})(g) = \sum_{i=1}^r \frac{|C_{CO_1}(g)|}{|C_{\bar{G}}(x_i)|}, \end{cases} \tag{1}$$

where $\{x_1, x_2, \cdots, x_r\}$ is the set of class representatives of $\bar{G}$ that fuse to $g$ in $Co_1$. Thus knowledge of the permutation character values on the classes of $Co_1$ will assist to know the possibilities for the fusion of classes of $\bar{G}$ to classes of $Co_1$. On the class where the permutation character value is zero, then no class of $\bar{G}$ fuses to it.
Let $[g_1]$ and $[g_2]$ be two arbitrary conjugacy classes of $Co_1$ such that $g_1^p \in [g_2]$ for some prime $p$. Let $[\bar{g}_1]$ and $[\bar{g}_2]$ be two arbitrary conjugacy classes of $\bar{G}$ such that $\bar{g}_1^p \in [\bar{g}_2]$ for some prime $p$. If the class $[\bar{g}_1]$ fuses to $[g_1]$ then clearly the class $[\bar{g}_2]$ must fuse to $[g_2]$. Thus knowledge of the power maps is also important in producing partial fusions and removing some ambiguities. The power maps of elements of $Co_1$ and its permutation character of degree 8292375 are given in the Atlas [11]. We give the permutation character below:

$$\chi(Co_1|\bar{G}) = 1\chi_1 + 17250\chi_6 + 80730\chi_{10} + 644644\chi_{16} + 2055625\chi_{25} + 5494125\chi_{32}.$$

Using the information provided by the conjugacy classes of elements of $2^{11}{:}M_{24}$ and $Co_1$ in the Atlas, the power maps and the permutation character of $Co_1$ of degree 8292375, we are able to obtain all the fusions of $\bar{G}$ into $Co_1$.

For example, obtaining the Character Table of $Co_1$ in GAP, we see that the character value of the identity class $1A$ of $Co_1$ is:

$$\chi_1 + \chi_6 + \chi_{10} + \chi_{16} + \chi_{25} + \chi_{32} = 1 + 17250 + 80730 + 644644 + 2055625 + 5494125 = 8292375.$$

Again with the help of GAP, we see that $|C_{Co_1}(1A)| = 4157776806543360000$ and $|C_{\bar{G}}(1A)| = 501397585920$. Now,

$$\frac{|C_{Co_1}(1A)|}{|C_{\bar{G}}(1A)|} = \frac{4157776806543360000}{501397585920} = 8292375.$$

This shows that $1A$ of $\bar{G}$ fuses into $1A$ of $Co_1$. Similarly, the character value of $2A$ of $Co_1$ is

$$\chi_1 + \chi_6 + \chi_{10} + \chi_{16} + \chi_{25} + \chi_{32} = 1 + 610 + 1626 + 3108 + 7625 + 19565 = 32535 \text{ and } |C_{Co_1}(2A)| = 89181388800.$$

We also have that $|C_{\bar{G}}(2A)| = 660602880$ and $|C_{\bar{G}}(2C)| = 2752512$. Thus,

$$\frac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2A)|} = \frac{89181388800}{660602880} = 135 \text{ and } \frac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2C)|} = \frac{89181388800}{2752512} = 32400.$$

We see that $\dfrac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2A)|} + \dfrac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2C)|} = 135 + 32400 = 32535$, which is the character value of $2A$ of $Co_1$ we found. This means that both $2A$ and $2C$ of $\bar{G}$ fuse into $2A$ of $Co_1$. Carrying out this operation on all the conjugacy classes of $\bar{G}$, we are able to identify which classes of $Co_1$ they fuse to. However, in some cases, having only the information about the conjugacy classes, power maps, permutation character values, may not be enough to complete the fusion map of certain classes due to some possibilities that arise. To obtain the full fusions in such cases, we restrict irreducible characters of the mother group of small degree to $\bar{G}$. In determining the restrictions of irreducible characters of the mother group to $\bar{G}$, we apply the technique of set intersections for characters. The concept of set intersections for characters has not been demonstrated in this paper but for more information about this concept (intersection of characters), readers are refered to [6], [7], [9] and [13].

## 7. GAP Programmes and Tables

The following are GAP programmes and Tables that have been cited in the main work.

**Programme A**

```
work :=[];;
output:=[];;
work[1]:=input[1];;
work[2]:=input[2];;
work[3]:=work[1] * work[2];;
work[4]:=work[3] * work[2];;
work[5]:=work[3] * work[4];;
work[6]:=work[3] * work[5];;
work[7]:=work[6] * work[3];;
work[8]:= work[6] * work[7];;
work[7]:=work[8] * work[8];;
work[2]:= work[7] * work[7];;
work[6]:=work[5] * work[5];;
work[1] := work[5] * work[6];;
work[5]:= work[4] * work[4];;
work[6] := work[4] * work[5];;
work[7]:= work[5] * work[6];;
work[8] := work[7]^-1;;
work[6] := work[8] * work[2];;
work[2] := work[6] * work[7];;
work[4] := work[3] * work[3];;
work[5] := work[3] * work[4];;
work[6] := work[5] * work[5];;
```

```
work[7] := work[6]^-1;;
work[8] := work[7] * work[1];;
work[1] := work[8] * work[6];;
output[1] := work[1];;
output[2] := work[2];;
```

**Programme D**

```
C:=List(ConjugacyClasses(G),Representative);;
M:=[]; g:=C[i];
for n in N do
Add(M, n*g*Inverse(n)*Inverse(g)); od;
M:=AsGroup(M);
cent:=Centralizer(G, g);;
O:=RightCosets(N,M);; D:=O;; B:=[];
for j in [1..Size(N)] do B[j]:=[];od; j:=1; while D <> [] do
x:=Representative(D[1]); for i in [1..Size(O)] do y:=x^cent; if
Intersection(y, O[i]) <> [] then Add(B[j],O[i]); fi;od;
D:=Difference(D,B[j]); j:=j+1; od; i:=1; while B[i] <> [] do
Print(Size(B[i]));Print(" - " );i:=i+1;
od;
I:=Irr(N);; IM:=[]; for i in [1..Size(I)] do if
IsSubgroup(Kernel(I[i]), M) then Add(IM,I[i]); fi; od;
oo:=Orbits(cent,IM);;
FM:=[];;
for i in [1..Size(oo)] do
Append(FM,[AsList(Sum(oo[i]))]);
od;
M1:=TransposedMat(FM);; M2:=AsDuplicateFreeList(M1);;
FM:=TransposedMat(M2);; Display(FM);
```

**Programme E**

```
K:=function()local K;;
K:=rec();;
K.SizesCentralizers:=[];;
K.OrdersClassRepresentatives:=[];;
K.Irr:=[[ ],[ ],[ ],...[ ]];;
K.UnderlyingCharacteristic:=0;;
ConvertToCharacterTable(K);;
return K;;
end;K:=K();;
SetInfoLevel(InfoCharacterTable,2);;
IsInternallyConsistent(K);
true
PossiblePowerMaps(K,p);
```

**Programme F**

```
for j in [1..4] do
for k in [2..4] do
for l in [3..4] do
for m in [4..4] do
if c'*gens[1]*Inverse(c')=gens[j] then
Print(j, " ");
elif c'*gens[1]*Inverse(c')=gens[j]*gens[k] then
Print(j, " ");
Print(k, " ");
```

```
elif c'*gens[1]*Inverse(c')=gens[j]*gens[k]*gens[l] then
Print(j, " ");
Print(k, " ");
Print(l, " ");
elif c'*gens[1]*Inverse(c')=gens[j]*gens[k]*gens[l]*gens[m] then
Print (j, " ");
Print (k, " ");
Print (l, " ");
Print (m, " ");
fi;
od;od;od;od;od;
```

**Programme G**

```
I:=Irr(N);;
o:=Orbits(G,I);;
M:=[AsList(Sum(o[1])),AsList(Sum(o[2]))\cdots,AsList(Sum(o[Size(o)]))];;
M1:=TransposedMat(M);;
M2:=AsDuplicateFreeList(M1);;
FM:=TransposedMat(M2);;
```

Table 1: Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$

| $[g]_G$ | $k$ | $f_j$ | $m_i$ | $[g]_{\bar{G}}$ | $\lvert C_{\bar{G}}(g) \rvert$ | 2 | 3 | 5 | 7 | Size of $[g]_{\bar{G}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $1A$ | 2048 | $f_1 = 1$ | $m_1 = 1$ | $1A$ | 501397585920 | $1A$ | $1A$ | $1A$ | $1A$ | 1 |
| | | $f_2 = 759$ | $m_2 = 759$ | $2A$ | 660602880 | $1A$ | $2A$ | $2A$ | $2A$ | 759 |
| | | $f_3 = 1288$ | $m_3 = 1288$ | $2B$ | 389283840 | $1A$ | $2B$ | $2B$ | $2B$ | 1288 |
| $2A$ | 64 | $f_1 = 1$ | $m_1 = 32$ | $2E$ | 491520 | $1A$ | $2E$ | $2E$ | $2E$ | 1020096 |
| | | $f_2 = 32$ | $m_2 = 1024$ | $2F$ | 15360 | $1A$ | $2F$ | $2F$ | $2F$ | 32643072 |
| | | $f_3 = 15$ | $m_3 = 480$ | $4E$ | 32768 | $2A$ | $4E$ | $4E$ | $4E$ | 15301440 |
| | | $f_4 = 15$ | $m_4 = 480$ | $4F$ | 32768 | $2A$ | $4F$ | $4F$ | $4F$ | 15301440 |
| | | $f_5 = 1$ | $m_5 = 32$ | $4G$ | 491520 | $2B$ | $4G$ | $4G$ | $4G$ | 1020096 |
| $2B$ | 128 | $f_1 = 1$ | $m_1 = 16$ | $2C$ | 344064 | $1A$ | $2C$ | $2C$ | $2C$ | 1457280 |
| | | $f_2 = 56$ | $m_2 = 896$ | $2D$ | 6144 | $1A$ | $2D$ | $2D$ | $2D$ | 81607680 |
| | | $f_3 = 56$ | $m_3 = 896$ | $4A$ | 6144 | $2A$ | $4A$ | $4A$ | $4A$ | 81607680 |
| | | $f_4 = 1$ | $m_4 = 16$ | $4B$ | 344064 | $2A$ | $4B$ | $4B$ | $4B$ | 1457280 |
| | | $f_5 = 7$ | $m_5 = 112$ | $4C$ | 49152 | $2A$ | $4C$ | $4C$ | $4C$ | 10200960 |
| | | $f_6 = 7$ | $m_6 = 112$ | $4D$ | 49152 | $2A$ | $4D$ | $4D$ | $4D$ | 10200960 |
| $3A$ | 32 | $f_1 = 1$ | $m_1 = 64$ | $3A$ | 34560 | $3A$ | $1A$ | $3A$ | $3A$ | 14508032 |
| | | $f_2 = 15$ | $m_2 = 960$ | $6A$ | 2304 | $3A$ | $2A$ | $6A$ | $6A$ | 217620480 |
| | | $f_3 = 10$ | $m_3 = 640$ | $6B$ | 3456 | $3A$ | $2A$ | $6B$ | $6B$ | 145080320 |
| | | $f_4 = 6$ | $m_4 = 384$ | $6C$ | 5760 | $3A$ | $2B$ | $6C$ | $6C$ | 87048192 |
| $3B$ | 8 | $f_1 = 1$ | $m_1 = 256$ | $3B$ | 4032 | $3B$ | $1A$ | $3B$ | $3B$ | 124354560 |
| | | $f_2 = 7$ | $m_2 = 896$ | $6D$ | 576 | $3B$ | $2B$ | $6D$ | $6D$ | 870481920 |
| $4A$ | 16 | $f_1 = 1$ | $m_1 = 128$ | $4K$ | 2048 | $2C$ | $8C$ | $4K$ | $4K$ | 244823040 |
| | | $f_2 = 2$ | $m_2 = 256$ | $4L$ | 1024 | $2C$ | $4K$ | $4L$ | $4L$ | 489646080 |
| | | $f_3 = 8$ | $m_3 = 1024$ | $8D$ | 256 | $4B$ | $8D$ | $8D$ | $8D$ | 1958584320 |

Table 1: Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$ (continued)

| $[g]_G$ | $k$ | $f_j$ | $m_i$ | $[g]_{\bar{G}}$ | $|C_{\bar{G}}(g)|$ | 2 | 3 | 5 | 7 | Size of $[g]_{\bar{G}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $f_4 = 1$ | $m_4 = 128$ | $8E$ | 2048 | $4B$ | $8E$ | $8E$ | $8E$ | 244823040 |
| | | $f_5 = 2$ | $m_5 = 256$ | $4M$ | 1024 | $2D$ | $4M$ | $4M$ | $4M$ | 489646080 |
| | | $f_6 = 1$ | $m_5 = 128$ | $8F$ | 2048 | $4A$ | $8F$ | $8F$ | $8F$ | 244823040 |
| | | $f_7 = 1$ | $m_5 = 128$ | $8G$ | 2048 | $4D$ | $8G$ | $8G$ | $8G$ | 244823040 |
| $4B$ | 8 | $f_1 = 1$ | $m_1 = 256$ | $4N$ | 768 | $2E$ | $4N$ | $4N$ | $4N$ | 652861440 |
| | | $f_2 = 3$ | $m_2 = 768$ | $8H$ | 256 | $4E$ | $8H$ | $8H$ | $8H$ | 1958584320 |
| | | $f_3 = 3$ | $m_3 = 768$ | $4O$ | 256 | $2F$ | $4O$ | $4O$ | $4O$ | 1958584320 |
| | | $f_4 = 1$ | $m_4 = 256$ | $8I$ | 768 | $4F$ | $8I$ | $8I$ | $8I$ | 652861440 |
| $4C$ | 16 | $f_1 = 1$ | $m_1 = 128$ | $4H$ | 6144 | $2C$ | $4H$ | $4H$ | $4H$ | 81607680 |
| | | $f_2 = 6$ | $m_2 = 768$ | $4I$ | 1024 | $2C$ | $4I$ | $4I$ | $4I$ | 489646080 |
| | | $f_3 = 1$ | $m_3 = 128$ | $8A$ | 6144 | $4A$ | $8A$ | $8A$ | $8A$ | 81607680 |
| | | $f_4 = 6$ | $m_2 = 768$ | $8B$ | 1024 | $4A$ | $8B$ | $8B$ | $8B$ | 489646080 |
| | | $f_5 = 1$ | $m_2 = 128$ | $4J$ | 6144 | $2D$ | $4J$ | $4J$ | $4J$ | 81607680 |
| | | $f_6 = 1$ | $m_2 = 128$ | $8C$ | 6144 | $4B$ | $8C$ | $8C$ | $8C$ | 81607680 |
| $5A$ | 8 | $f_1 = 1$ | $m_1 = 256$ | $5A$ | 2480 | $5A$ | $5A$ | $1A$ | $5A$ | 2021764459 |
| | | $f_2 = 3$ | $m_2 = 768$ | $10A$ | 160 | $5A$ | $10A$ | $2A$ | $10A$ | 3133734912 |
| | | $f_3 = 4$ | $m_3 = 1024$ | $10B$ | 120 | $5A$ | $10B$ | $2B$ | $10B$ | 4178313216 |
| $6A$ | 4 | $f_1 = 1$ | $m_1 = 512$ | $6G$ | 96 | $3B$ | $2E$ | $6G$ | $6G$ | 5222891520 |
| | | $f_2 = 2$ | $m_2 = 1024$ | $6H$ | 48 | $3B$ | $2F$ | $6H$ | $6H$ | 10445783040 |
| | | $f_3 = 1$ | $m_3 = 512$ | $12E$ | 96 | $6D$ | $4G$ | $12E$ | $12E$ | 10445783040 |
| $6B$ | 8 | $f_1 = 1$ | $m_1 = 256$ | $6E$ | 192 | $3A$ | $2C$ | $6E$ | $6E$ | 5222891520 |
| | | $f_2 = 2$ | $m_2 = 512$ | $6F$ | 96 | $3A$ | $2D$ | $6F$ | $6F$ | 2611445760 |
| | | $f_3 = 1$ | $m_3 = 256$ | $12A$ | 192 | $6A$ | $4A$ | $12A$ | $12A$ | 5222891520 |
| | | $f_4 = 2$ | $m_2 = 512$ | $12B$ | 96 | $6A$ | $4B$ | $12B$ | $12B$ | 2611445760 |
| | | $f_5 = 1$ | $m_3 = 256$ | $12C$ | 192 | $6A$ | $4C$ | $12C$ | $12C$ | 5222891520 |
| | | $f_6 = 1$ | $m_3 = 256$ | $12D$ | 192 | $6A$ | $4D$ | $12D$ | $12D$ | 5222891520 |
| $7A$ | 4 | $f_1 = 1$ | $m_1 = 512$ | $7A$ | 168 | $7A$ | $7B$ | $7B$ | $1A$ | 2984509440 |
| | | $f_2 = 3$ | $m_2 = 1536$ | $14A$ | 56 | $7A$ | $14B$ | $14B$ | $2A$ | 8953528320 |
| $7B$ | 4 | $f_1 = 1$ | $m_1 = 512$ | $7B$ | 168 | $7B$ | $7A$ | $7A$ | $1A$ | 2984509440 |
| | | $f_2 = 3$ | $m_2 = 1536$ | $14B$ | 56 | $7B$ | $14A$ | $14A$ | $2A$ | 8953528320 |
| $8A$ | 4 | $f_1 = 1$ | $m_1 = 128$ | $8J$ | 256 | $4K$ | $8J$ | $8J$ | $8J$ | 1958584320 |
| | | $f_2 = 1$ | $m_2 = 128$ | $8K$ | 128 | $4L$ | $8K$ | $8K$ | $8K$ | 1958584320 |
| | | $f_3 = 1$ | $m_3 = 128$ | $16A$ | 32 | $8E$ | $16A$ | $16A$ | $16A$ | 1958584320 |
| | | $f_4 = 1$ | $m_4 = 128$ | $16B$ | 256 | $1A$ | $16B$ | $16B$ | $16B$ | 1958584320 |
| $10A$ | 4 | $f_1 = 1$ | $m_1 = 512$ | $10C$ | 80 | $5A$ | $10C$ | $2E$ | $10C$ | 6267469824 |
| | | $f_2 = 2$ | $m_2 = 1024$ | $10D$ | 40 | $5A$ | $10D$ | $2F$ | $10D$ | 12534939650 |
| | | $f_3 = 1$ | $m_3 = 512$ | $20A$ | 80 | $10B$ | $20A$ | $4G$ | $20A$ | 6267469824 |
| $11A$ | 2 | $f_1 = 1$ | $m_1 = 1024$ | $11A$ | 22 | $11A$ | $11A$ | $11A$ | $11A$ | 22790799360 |
| | | $f_2 = 1$ | $m_2 = 1024$ | $22A$ | 22 | $11A$ | $22A$ | $22A$ | $22A$ | 22790799360 |

Table 1: Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$ (continued)

| $[g]_G$ | $k$ | $f_j$ | $m_i$ | $[g]_{\bar{G}}$ | $\lvert C_{\bar{G}}(g)\rvert$ | 2 | 3 | 5 | 7 | Size of $[g]_{\bar{G}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 12A | 4 | $f_1 = 1$ | $m_1 = 512$ | 12F | 48 | 6E | 4H | 12F | 12A | 10445783040 |
| | | $f_2 = 1$ | $m_2 = 512$ | 24A | 48 | 12A | 8B | 24A | 24A | 10445783040 |
| | | $f_3 = 1$ | $m_3 = 512$ | 12G | 48 | 6E | 4I | 12G | 12G | 10445783040 |
| | | $f_4 = 1$ | $m_4 = 512$ | 24B | 48 | 12A | 8A | 24B | 24B | 10445783040 |
| 12B | 2 | $f_1 = 1$ | $m_1 = 1024$ | 12H | 24 | 6G | 4N | 12H | 12H | 20981566080 |
| | | $f_2 = 1$ | $m_2 = 1024$ | 12I | 24 | 6H | 4O | 12I | 12I | 20981566080 |
| 14A | 2 | $f_1 = 1$ | $m_1 = 1024$ | 14C | 28 | 7A | 14D | 14D | 2C | 17907056640 |
| | | $f_2 = 1$ | $m_2 = 1024$ | 28A | 28 | 14A | 28B | 28B | 4A | 17907056640 |
| 14B | 2 | $f_1 = 1$ | $m_1 = 1024$ | 14D | 28 | 7B | 14C | 14C | 2C | 17907056640 |
| | | $f_2 = 1$ | $m_2 = 1024$ | 28B | 28 | 14B | 28A | 28A | 4A | 17907056640 |
| 15A | 2 | $f_1 = 1$ | $m_1 = 1024$ | 15A | 30 | 15A | 5A | 3A | 15B | 16713252860 |
| | | $f_2 = 1$ | $m_2 = 1024$ | 30A | 30 | 15A | 10A | 6B | 30B | 16713252860 |
| 15B | 2 | $f_1 = 1$ | $m_1 = 1024$ | 15B | 30 | 15B | 5A | 3A | 15A | 16713252860 |
| | | $f_2 = 1$ | $m_2 = 1024$ | 30B | 30 | 15B | 10A | 6B | 30A | 16713252860 |
| 21A | 1 | $f_1 = 1$ | $m_1 = 2048$ | 21A | 21 | 21A | 7B | 21B | 3B | 23876075520 |
| 21B | 1 | $f_1 = 1$ | $m_1 = 2048$ | 21B | 21 | 21B | 7A | 21A | 3B | 23876075520 |
| 23A | 1 | $f_1 = 1$ | $m_1 = 2048$ | 23A | 23 | 23A | 23A | 23B | 23B | 21799895040 |
| 23B | 1 | $f_1 = 1$ | $m_1 = 2048$ | 23B | 23 | 23B | 23B | 23A | 23A | 21799895040 |

Table 2: Fusion of the Classes from $H_2$ to $G = M_{24}$

| $[h]_{H_2} \mapsto [g]_G$ | | $[h]_{H_2} \mapsto [g]_G$ | | $[h]_{H_2} \mapsto [g]_G$ | | $[h]_{H_2} \mapsto [g]_G$ | |
|---|---|---|---|---|---|---|---|
| 1a | 1A | 4a | 4A | 6a | 6B | 8b | 8A |
| 2c | 2A | 4b | 4A | 6b | 6B | 10a | 10a |
| 2a | 2B | 4d | 4A | 7a | 7A | 11a | 11A |
| 2b | 2B | 4c | 4C | 7b | 7B | 12a | 12A |
| 3a | 3A | 5a | 5A | 8a | 8A | 14a | 14A |

Table 3: Fusion of the Classes from $H_3$ to $G = M_{24}$

| $[h]_{H_3} \mapsto [g]_G$ | | $[h]_{H_3} \mapsto [g]_G$ | | $[h]_{H_3} \mapsto [g]_G$ | | $[h]_{H_3} \mapsto [g]_G$ | | $[h]_{H_3} \mapsto [g]_G$ | | $[h]_{H_3} \mapsto [g]_G$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1a | 1A | 2c | 2B | 4a | 4A | 4i | 4B | 5A | 5A | 6c | 6B |
| 2b | 2A | 2e | 2B | 4c | 4A | 4b | 4C | 6d | 6A | 8a | 8A |
| 2d | 2A | 3a | 3A | 4f | 4A | 4d | 4C | 6e | 6A | 10a | 10A |
| 2f | 2A | 3b | 3A | 4e | 4B | 4g | 4C | 6a | 6B | 12a | 12A |
| 2a | 2B | 3c | 3B | 4h | 4B | 4j | 4C | 6b | 6B | 12b | 12A |
| 12c | 12B | 15b | 15A | 15a | 15B | | | | | | |

Table 4: Character Table $H_1 = G = M_{24}$

| $[g]$ | 1A | 3A | 5A | 15A | 15B | 2A | 10A | 3B | 6A | 2B | 4A | 8A | 4B | 7A | 7B | 21A | 21B | 14A | 14B | 4C | 6B | 12A | 11A | 23A | 23B | 12B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| order | | | | 15 | 15 | | 20 | | 24 | | | 16 | | 42 | 42 | 21 | 21 | 14 | 14 | 384 | 24 | 12 | 11 | 23 | 23 | 12 |
| $\lvert C_{H_1}(g)\rvert$ | 244823040 | 1080 | 60 | 15 | 15 | 7680 | 20 | 504 | 24 | 21504 | 128 | 16 | 96 | 42 | 42 | 21 | 21 | 14 | 14 | 384 | 24 | 12 | 11 | 23 | 23 | 12 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 23 | 5 | 3 | 0 | 0 | -1 | -1 | -1 | -1 | 7 | 3 | 1 | -1 | 2 | 2 | -1 | -1 | 0 | 0 | -1 | 1 | -1 | 1 | 0 | 0 | -1 |
| $\chi_3$ | 45 | 0 | 0 | 0 | 0 | 5 | 0 | 3 | -1 | -3 | 1 | -1 | 1 | $B$ | $/B$ | $/B$ | $B$ | $-/B$ | $-B$ | -3 | 0 | 0 | 1 | -1 | -1 | 1 |
| $\chi_4$ | 45 | 0 | 0 | 0 | 0 | 5 | 0 | 3 | -1 | -3 | 1 | -1 | 1 | $/B$ | $B$ | $B$ | $/B$ | $-B$ | $-/B$ | -3 | 0 | 0 | 1 | -1 | -1 | 1 |
| $\chi_5$ | 231 | -3 | 1 | $A$ | $/A$ | -9 | 1 | 0 | 0 | 7 | -1 | -1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | -1 | 0 | 1 | 1 | 1 |
| $\chi_6$ | 231 | -3 | 1 | $/A$ | $A$ | -9 | 1 | 0 | 0 | 7 | -1 | -1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | -1 | 0 | 1 | 1 | 1 |
| $\chi_7$ | 252 | 9 | 2 | -1 | -1 | 12 | 2 | 0 | 0 | 28 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 1 | -1 | -1 | 0 | 0 | 0 |
| $\chi_8$ | 253 | 10 | 3 | 0 | 0 | -11 | -1 | 1 | 1 | 13 | 1 | -1 | 1 | 0 | 0 | 1 | 1 | -1 | -1 | -3 | -2 | -1 | 0 | 0 | 0 | 0 |
| $\chi_9$ | 483 | 6 | -2 | 1 | 1 | 3 | -2 | 0 | 0 | 35 | 3 | -1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 2 | 0 | -1 | 0 | 0 | 0 |
| $\chi_{10}$ | 770 | 5 | 0 | 0 | 0 | 10 | 0 | -7 | 1 | -14 | -2 | 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | $D$ | $/D$ | 1 |
| $\chi_{11}$ | 770 | 5 | 0 | 0 | 0 | 10 | 0 | -7 | 1 | -14 | -2 | 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | $/D$ | $D$ | 1 |
| $\chi_{12}$ | 990 | 0 | 0 | 0 | 0 | -10 | 0 | 3 | -1 | -18 | 2 | 0 | -2 | $B$ | $/B$ | $/B$ | $B$ | $/B$ | $B$ | 6 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\chi_{13}$ | 990 | 0 | 0 | 0 | 0 | -10 | 0 | 3 | -1 | -18 | 2 | 0 | -2 | $/B$ | $B$ | $B$ | $/B$ | $B$ | $/B$ | 6 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\chi_{14}$ | 1035 | 0 | 0 | 0 | 0 | 35 | 0 | 6 | 2 | 27 | -1 | 1 | 3 | -1 | -1 | -1 | -1 | -1 | -1 | 3 | 0 | -1 | 1 | 0 | 0 | 0 |
| $\chi_{15}$ | 1035 | 0 | 0 | 0 | 0 | -5 | 0 | -3 | 1 | -21 | 3 | -1 | -1 | $C$ | $/C$ | $-/B$ | $-B$ | 0 | 0 | 3 | 0 | 0 | 1 | 0 | 0 | -1 |
| $\chi_{16}$ | 1035 | 0 | 0 | 0 | 0 | -5 | 0 | -3 | 1 | -21 | 3 | -1 | -1 | $/C$ | $C$ | $-B$ | $-/B$ | 0 | 0 | 3 | 0 | 0 | 1 | 0 | 0 | -1 |
| $\chi_{17}$ | 1265 | 5 | 1 | 1 | 1 | -15 | 0 | 8 | 0 | 49 | 1 | 1 | -3 | -2 | -2 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 0 |
| $\chi_{18}$ | 1771 | 16 | 1 | 1 | 1 | 11 | 1 | 7 | -1 | -21 | -5 | -1 | -1 | 0 | 0 | 0 | 0 | 1 | 1 | -7 | 1 | 0 | 1 | 0 | 0 | 0 |
| $\chi_{19}$ | 2024 | -1 | -1 | -1 | -1 | 24 | -1 | 8 | 0 | 8 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | -1 |
| $\chi_{20}$ | 2277 | 0 | -3 | 0 | 0 | -19 | 1 | 6 | 2 | 21 | 1 | -1 | -3 | 2 | 2 | -1 | -1 | 0 | 0 | -3 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{21}$ | 3312 | 0 | -3 | 0 | 0 | 16 | 1 | -6 | -2 | 48 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $\chi_{22}$ | 3520 | 10 | 0 | 0 | 0 | 0 | 0 | -8 | 0 | 64 | 0 | 0 | 0 | -1 | -1 | -1 | -1 | 1 | 1 | 0 | -2 | 0 | 0 | 1 | 1 | 0 |
| $\chi_{23}$ | 5313 | -15 | 3 | 0 | 0 | 9 | -1 | 0 | 0 | 49 | -3 | -1 | -3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{24}$ | 5544 | 9 | -1 | -1 | -1 | 24 | -1 | 0 | 0 | -56 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -8 | 1 | 1 | 0 | 1 | 1 | 0 |
| $\chi_{25}$ | 5796 | -9 | 1 | 1 | 1 | 36 | 1 | 0 | 0 | -28 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -4 | -1 | 1 | -1 | 0 | 0 | 0 |
| $\chi_{26}$ | 10395 | 0 | 0 | 0 | 0 | -45 | 0 | 0 | 0 | -21 | -1 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | -1 | -1 | 0 |

Table 5: Character Table of $H_2 = M_{22}{:}2$

| $[g]$ | 1a | 2a | 3a | 4a | 6a | 12a | 4b | 2b | 4c | 8a | 4d | 8b | 7a | 7b | 2c | 14a | 14b | 5a | 10a | 11a | 6b |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|C_{H_2}(g)\|$ | 887040 | 768 | 72 | 64 | 24 | 12 | 64 | 7680 | 96 | 16 | 32 | 16 | 14 | 14 | 640 | 14 | 14 | 10 | 10 | 11 | 12 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | −1 | 1 | 1 | 1 | −1 | −1 | −1 | 1 | −1 | 1 | −1 |
| $\chi_3$ | 21 | 5 | 3 | −1 | −1 | −1 | 1 | −1 | 1 | 1 | 3 | −1 | 0 | 0 | 7 | 0 | 0 | 1 | −1 | −1 | 1 |
| $\chi_4$ | 21 | 5 | 3 | 1 | −1 | 1 | 1 | 1 | 1 | −1 | −3 | −1 | 0 | 0 | −7 | 0 | 0 | 1 | 1 | −1 | −1 |
| $\chi_5$ | 45 | −3 | 0 | −3 | 0 | 0 | 1 | 5 | 1 | −1 | 1 | −1 | $A$ | $/A$ | −3 | $-A$ | $-/A$ | 0 | 0 | 1 | 0 |
| $\chi_6$ | 45 | −3 | 0 | −3 | 0 | 0 | 1 | 5 | 1 | −1 | 1 | −1 | $/A$ | $A$ | −3 | $-/A$ | $-A$ | 0 | 0 | 1 | 0 |
| $\chi_7$ | 45 | −3 | 0 | 3 | 0 | 0 | 1 | −5 | 1 | 1 | −1 | −1 | $A$ | $/A$ | 3 | $A$ | $/A$ | 0 | 0 | 1 | 0 |
| $\chi_8$ | 45 | −3 | 0 | 3 | 0 | 0 | 1 | −5 | 1 | 1 | −1 | −1 | $/A$ | $A$ | 3 | $/A$ | $A$ | 0 | 0 | 1 | 0 |
| $\chi_9$ | 55 | 7 | 1 | 1 | 1 | 1 | −1 | 5 | 3 | −1 | 1 | 1 | −1 | −1 | 13 | −1 | −1 | 0 | 0 | 0 | 1 |
| $\chi_{10}$ | 99 | 3 | 0 | 3 | 0 | 0 | −1 | −1 | 3 | −1 | −1 | −1 | 1 | 1 | 15 | 1 | 1 | 0 | 0 | 0 | −1 |
| $\chi_{11}$ | 99 | 3 | 0 | −3 | 0 | 0 | −1 | 1 | 3 | 1 | 1 | −1 | 1 | 1 | −15 | −1 | −1 | −1 | −1 | 0 | 0 |
| $\chi_{12}$ | 154 | 10 | 1 | 2 | 1 | −1 | 2 | 6 | −2 | 0 | 2 | 0 | 0 | 0 | 14 | 0 | 0 | −1 | 1 | 0 | 0 |
| $\chi_{13}$ | 154 | 10 | 1 | −2 | 1 | 1 | 2 | −6 | −2 | 0 | −2 | 0 | 0 | 0 | −14 | 0 | 0 | −1 | 1 | 0 | −1 |
| $\chi_{14}$ | 210 | 2 | 3 | −2 | −1 | 1 | −2 | −10 | −2 | 0 | 2 | 0 | 0 | 0 | 14 | 0 | 0 | −1 | −1 | 0 | 1 |
| $\chi_{15}$ | 210 | 2 | 3 | 2 | −1 | −1 | −2 | 10 | −2 | 0 | −2 | 0 | 0 | 0 | −14 | 0 | 0 | 0 | 0 | 1 | −1 |
| $\chi_{16}$ | 231 | 7 | −3 | −1 | 1 | −1 | −1 | −9 | −1 | −1 | −1 | −1 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 1 | 1 |
| $\chi_{17}$ | 231 | 7 | −3 | −1 | 1 | −1 | −1 | −9 | −1 | −1 | −1 | −1 | 0 | 0 | 7 | 0 | 0 | 1 | 1 | 0 | 1 |
| $\chi_{18}$ | 231 | 7 | −3 | 1 | 1 | 1 | −1 | 9 | −1 | 1 | 1 | −1 | 0 | 0 | −7 | 0 | 0 | 1 | −1 | 0 | −1 |
| $\chi_{19}$ | 385 | 1 | −2 | −3 | −2 | 0 | 1 | 5 | 1 | 1 | −3 | 1 | 0 | 0 | 21 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{20}$ | 385 | 1 | −2 | 3 | −2 | 0 | 1 | −5 | 1 | −1 | 3 | 1 | 0 | 0 | −21 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{21}$ | 560 | −16 | 2 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 |

In Tables 4, 5 and 6, we take $A = \dfrac{-1-\sqrt{-15}}{2}, B = \dfrac{-1-\sqrt{-7}}{2}, C = -1-\sqrt{-7}$ and $D = \dfrac{-1-\sqrt{-23}}{2}$, where $/A, /B, /C$ and $/D$ are conjugates of $A, B, C$ and $D$ respectively.

Table 6: Character Table of $H_3 = 2^6{:}(3.A_6){:}2$

| $[g]$ | 1a | 2a | 2b | 3a | 3b | 6a | 4a | 2c | 4b | 2d | 6b | 10a | 5a | 15a | 15b | 4c | 2e | 4d | 8a | 4e | 4f | 12a | 6c | 12b | 4g | 4h | 6d | 3c | 4i | 4j | 2f | 6e | 12c |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|C_{H_3}(g)\|$ | 138240 | 3072 | 7680 | 1080 | 72 | 24 | 128 | 768 | 128 | 256 | 24 | 20 | 60 | 15 | 15 | 64 | 384 | 384 | 16 | 32 | 32 | 12 | 12 | 12 | 96 | 32 | 24 | 72 | 96 | 128 | 384 | 12 | 12 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_3$ | 5 | 5 | 5 | 5 | $-1$ | $-1$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 1 | $-1$ | $-1$ | $-1$ | $-1$ | 2 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_4$ | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | $-3$ | $-3$ | 1 | $-1$ | $-1$ | $-1$ | 1 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 2 | 1 | $-3$ | $-3$ | 0 | 0 |
| $\chi_5$ | 5 | 5 | 5 | 5 | $-1$ | $-1$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | $-3$ | $-1$ | $-1$ | $-1$ | 1 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 2 | 1 | 3 | 1 | 0 | 1 |
| $\chi_6$ | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | $-1$ | 3 | 3 | 3 | 0 | 0 |
| $\chi_7$ | 6 | 6 | 6 | $-3$ | 0 | 0 | $-2$ | $-2$ | $-2$ | $-2$ | 1 | 1 | 1 | $A$ | $/A$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | 0 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_8$ | 6 | 6 | 6 | $-3$ | 0 | 0 | $-2$ | $-2$ | $-2$ | $-2$ | 1 | 1 | 1 | $/A$ | $A$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | 0 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_9$ | 9 | 9 | 9 | 9 | 0 | 0 | 1 | 1 | 1 | 1 | $-2$ | $-1$ | $-1$ | $-1$ | $-1$ | $-3$ | $-3$ | $-3$ | $-1$ | $-1$ | $-1$ | 0 | $-1$ | 0 | $-3$ | $-3$ | 0 | 0 | $-3$ | $-3$ | $-3$ | 0 | 0 |
| $\chi_{10}$ | 9 | 9 | 9 | 9 | 0 | 0 | 1 | 1 | 1 | 1 | $-2$ | $-1$ | $-1$ | $-1$ | $-1$ | 3 | 3 | 3 | 1 | 1 | 1 | 0 | 1 | 0 | 3 | 3 | 1 | 0 | 3 | 3 | 3 | 0 | 0 |
| $\chi_{11}$ | 10 | 10 | 10 | 10 | 1 | 1 | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | 0 | 0 | 0 | 0 | $-2$ | $-2$ | $-2$ | 0 | 0 | 0 | $-1$ | 0 | $-1$ | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 |
| $\chi_{12}$ | 10 | 10 | 10 | 10 | 1 | 1 | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | $-1$ | 0 | $-1$ | $-2$ | $-1$ | $-1$ | $-2$ | $-2$ | $-2$ | $-2$ | $-1$ | $-1$ |
| $\chi_{13}$ | 12 | 12 | 12 | $-6$ | 0 | 0 | 4 | 4 | 4 | 4 | $-2$ | 2 | 2 | $-1$ | $-1$ | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | 0 | $-1$ | 3 | 3 | 0 | 0 | 2 | 2 | 2 | 1 | 1 |
| $\chi_{14}$ | 16 | 16 | 16 | 16 | $-2$ | $-2$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 4 | 0 | 2 | 2 | 0 | 0 | 0 | 3 | 3 | 0 | $-2$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{15}$ | 18 | 2 | $-6$ | 0 | 3 | $-1$ | 2 | 6 | $-2$ | $-2$ | 0 | $-1$ | 3 | 0 | 0 | $-4$ | $-4$ | $-4$ | 0 | $-2$ | $-2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{16}$ | 18 | 2 | $-6$ | 0 | 3 | $-1$ | 2 | 6 | $-2$ | $-2$ | 0 | $-1$ | 3 | 0 | 0 | 4 | 4 | 4 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{17}$ | 18 | 18 | 18 | $-9$ | 0 | 0 | 2 | 2 | 2 | 2 | $-1$ | $-2$ | $-2$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{18}$ | 30 | 30 | 18 | $-15$ | 0 | 0 | $-2$ | $-2$ | $-2$ | $-2$ | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | $-1$ | 0 | 0 | $-1$ | $-1$ | $-1$ | $-2$ | $-2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{19}$ | 45 | $-3$ | 5 | 0 | 0 | 0 | $-3$ | 9 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 3 | 1 | 3 | 3 | 0 | 0 | 0 | 3 | $-1$ | $-1$ | 0 | $-1$ | $-1$ | 7 | 1 | 1 |
| $\chi_{20}$ | 45 | $-3$ | 5 | 0 | 0 | 0 | $-3$ | 9 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | $-3$ | $-3$ | $-1$ | $-3$ | $-3$ | 0 | 0 | 0 | $-3$ | 1 | $-1$ | 0 | 1 | 1 | $-7$ | $-1$ | $-1$ |
| $\chi_{21}$ | 45 | $-3$ | 5 | 0 | 0 | 0 | 1 | $-3$ | $-3$ | 5 | 0 | 0 | 0 | 0 | 0 | 1 | $-3$ | $-3$ | 1 | 1 | 1 | 0 | 0 | 0 | 3 | $-3$ | 1 | 0 | $-3$ | $-3$ | 5 | 1 | 1 |
| $\chi_{22}$ | 45 | $-3$ | 5 | 0 | 0 | 0 | 1 | $-3$ | $-3$ | 5 | 0 | 0 | 0 | 0 | 0 | $-1$ | 3 | 3 | $-1$ | $-1$ | $-1$ | 0 | 0 | 0 | $-3$ | 3 | $-1$ | 0 | 3 | 3 | $-5$ | $-1$ | $-1$ |
| $\chi_{23}$ | 72 | 8 | $-24$ | 0 | 3 | $-1$ | 0 | 0 | 0 | 0 | 0 | 1 | $-3$ | 0 | 0 | 0 | $-8$ | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{24}$ | 72 | 8 | $-24$ | 0 | 3 | $-1$ | 0 | 0 | 0 | 0 | 0 | 1 | $-3$ | 0 | 0 | 0 | 8 | $-8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{25}$ | 90 | $-6$ | 10 | 0 | 0 | 0 | $-2$ | 6 | $-2$ | 6 | 0 | 0 | 0 | 0 | 0 | $-2$ | 6 | 6 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | $-2$ | $-2$ | 2 | 0 | 0 |
| $\chi_{26}$ | 90 | $-6$ | 10 | 0 | 0 | 0 | $-2$ | 6 | $-2$ | 6 | 0 | 0 | 0 | 0 | 0 | 2 | $-6$ | $-6$ | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | 0 | 0 | $-1$ | 0 | 2 | 2 | $-2$ | 0 | 0 |
| $\chi_{27}$ | 90 | 10 | $-30$ | 0 | $-3$ | 1 | 2 | 6 | $-2$ | $-2$ | 0 | 0 | 0 | 0 | 0 | 0 | $-4$ | 4 | 0 | 2 | 2 | $-1$ | $-1$ | $-1$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{28}$ | 90 | 10 | $-30$ | 0 | $-3$ | 1 | 2 | 6 | $-2$ | $-2$ | 0 | 0 | 0 | 0 | 0 | 0 | 4 | $-4$ | 0 | $-2$ | $-2$ | 1 | 1 | 1 | 0 | 0 | $-1$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{29}$ | 108 | 12 | $-36$ | 0 | 0 | 0 | $-4$ | $-12$ | 4 | 4 | 0 | $-1$ | 3 | 0 | 0 | $-1$ | 3 | 3 | $-1$ | 1 | 1 | 1 | 1 | 1 | 3 | $-1$ | 1 | 0 | 3 | $-1$ | $-9$ | 1 | 1 |
| $\chi_{30}$ | 135 | $-9$ | 15 | 0 | 0 | 0 | $-1$ | 3 | 3 | $-5$ | 0 | 0 | 0 | 0 | 0 | 1 | $-3$ | $-3$ | $-1$ | $-1$ | $-1$ | 0 | 0 | 0 | $-3$ | 1 | 1 | 0 | $-3$ | $-1$ | 9 | 0 | 0 |
| $\chi_{31}$ | 135 | $-9$ | 15 | 0 | 0 | 0 | $-1$ | 3 | 3 | $-5$ | 0 | 0 | 0 | 0 | 0 | 1 | $-3$ | $-3$ | $-1$ | $-1$ | $-1$ | 0 | 0 | 0 | $-3$ | 1 | 1 | 0 | $-3$ | 5 | $-3$ | 0 | 0 |
| $\chi_{32}$ | 135 | $-9$ | 15 | 0 | 0 | 0 | 3 | $-9$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 | 0 | $-1$ | 3 | 3 | $-1$ | 1 | 1 | 0 | 0 | 0 | 3 | $-3$ | $-1$ | 0 | 3 | 5 | $-3$ | 0 | 0 |
| $\chi_{33}$ | 135 | $-9$ | 15 | 0 | 0 | 0 | 3 | $-9$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 | 0 | $-1$ | 3 | 3 | 1 | $-1$ | $-1$ | 0 | 0 | 0 | 3 | $-3$ | $-1$ | 0 | 3 | $-5$ | 3 | 0 | 0 |

Table 7: Fischer Matrices of $2^{11}{:}M_{24}$

| Fischer Matrices of $2^{11}{:}M_{24}$ | Fischer Matrices of $2^{11}{:}M_{24}$ |
|---|---|

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 276 & 20 & -12 \\ 1771 & -21 & 11 \end{pmatrix}$$

$$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 12 & 0 & -4 & -12 & 4 \\ 1 & -1 & 1 & 1 & 1 \\ 30 & 0 & -2 & 30 & -2 \\ 20 & 0 & 4 & -20 & -4 \end{pmatrix}$$

$$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 28 & 4 & -4 & -4 & -4 & -28 \\ 8 & 0 & 0 & 8 & -8 & -8 \\ 7 & -1 & -1 & 7 & 7 & 7 \\ 28 & -4 & 4 & -4 & -4 & -28 \\ 56 & 0 & 0 & -8 & 8 & 56 \end{pmatrix}$$

$$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 15 & -1 & 3 & -5 \\ 1 & 1 & -1 & -1 \\ 15 & -1 & -3 & 5 \end{pmatrix}$$

$$M(3B) = \begin{pmatrix} 1 & 1 \\ 7 & -1 \end{pmatrix}$$

$$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & -2 & 2 & -2 & 2 & -2 \\ 2 & 0 & -2 & -2 & 2 & 2 & 2 \\ 4 & 0 & 0 & 0 & -4 & -4 & 4 \\ 1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & -2 & -2 & 2 & -2 \\ 4 & 0 & 0 & 0 & 4 & -4 & -4 \end{pmatrix}$$

$$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -3 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 0 & -4 & 0 & 4 & -4 \\ 3 & -1 & 3 & -1 & 3 & 3 \\ 1 & -1 & 1 & 1 & -1 & -1 \\ 4 & 0 & -4 & 0 & -4 & 4 \\ 3 & 1 & 3 & -1 & -3 & -3 \end{pmatrix}$$

$$M(5A) = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 0 & -2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$M(6A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & -2 \end{pmatrix}$$

$$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & -2 & 2 & -2 & 0 & 0 \end{pmatrix}$$

$$M(7A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$M(7B) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

$$M(10A) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

$$M(11A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(12A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

$$M(12B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(14A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(14B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(15A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(15B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(21A) = (1)$$

$$M(21B) = (1)$$

$$M(23A) = (1)$$

$$M(23B) = (1)$$

Table 8: The Character Table of $\bar{G} = 2^{11}{:}M_{24}$

| $[g]_{\bar{G}}$ | 1A | 2A | 2B | 2E | 2F | 4E | 4F | 4G | 2C | 2D | 4A | 4B | 4C | 4D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $|C_{\bar{G}}(g)|$ | 501397585920 | 660602880 | 389283840 | 491520 | 15360 | 32768 | 32768 | 491520 | 344064 | 6144 | 6144 | 344064 | 49152 | 49152 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 23 | 23 | 23 | −1 | −1 | −1 | −1 | −1 | 7 | 7 | 7 | 7 | 7 | 7 |
| $\chi_3$ | 45 | 45 | 45 | 5 | 5 | 5 | 5 | 5 | −3 | −3 | −3 | −3 | −3 | −3 |
| $\chi_4$ | 45 | 45 | 45 | 5 | 5 | 5 | 5 | 5 | −3 | −3 | −3 | −3 | −3 | −3 |
| $\chi_5$ | 231 | 231 | 231 | −9 | −9 | −9 | −9 | −9 | 7 | 7 | 7 | 7 | 7 | 7 |
| $\chi_6$ | 231 | 231 | 231 | −9 | −9 | −9 | −9 | −9 | 7 | 7 | 7 | 7 | 7 | 7 |
| $\chi_7$ | 252 | 252 | 252 | 12 | 12 | 12 | 12 | 12 | 28 | 28 | 28 | 28 | 28 | 28 |
| $\chi_8$ | 253 | 253 | 253 | −11 | −11 | −11 | −11 | −11 | 13 | 13 | 13 | 13 | 13 | 13 |
| $\chi_9$ | 483 | 483 | 483 | 3 | 3 | 3 | 3 | 3 | 35 | 35 | 35 | 35 | 35 | 35 |
| $\chi_{10}$ | 770 | 770 | 770 | 10 | 10 | 10 | 10 | 10 | −14 | −14 | −14 | −14 | −14 | −14 |
| $\chi_{11}$ | 770 | 770 | 770 | 10 | 10 | 10 | 10 | 10 | −14 | −14 | −14 | −14 | −14 | −14 |
| $\chi_{12}$ | 990 | 990 | 990 | −10 | −10 | −10 | −10 | −10 | −18 | −18 | −18 | −18 | −18 | −18 |
| $\chi_{13}$ | 990 | 990 | 990 | −10 | −10 | −10 | −10 | −10 | −18 | −18 | −18 | −18 | −18 | −18 |
| $\chi_{14}$ | 1035 | 1035 | 1035 | 35 | 35 | 35 | 35 | 35 | 27 | 27 | 27 | 27 | 27 | 27 |
| $\chi_{15}$ | 1035 | 1035 | 1035 | −5 | −5 | −5 | −5 | −5 | −21 | −21 | −21 | −21 | −21 | −21 |
| $\chi_{16}$ | 1035 | 1035 | 1035 | −5 | −5 | −5 | −5 | −5 | −21 | −21 | −21 | −21 | −21 | −21 |
| $\chi_{17}$ | 1265 | 1265 | 1265 | −15 | −15 | −15 | −15 | −15 | 49 | 49 | 49 | 49 | 49 | 49 |
| $\chi_{18}$ | 1771 | 1771 | 1771 | 11 | 11 | 11 | 11 | 11 | −21 | −21 | −21 | −21 | −21 | −21 |
| $\chi_{19}$ | 2024 | 2024 | 2024 | 24 | 24 | 24 | 24 | 24 | 8 | 8 | 8 | 8 | 8 | 8 |
| $\chi_{20}$ | 2277 | 2277 | 2277 | −19 | −19 | −19 | −19 | −19 | 21 | 21 | 21 | 21 | 21 | 21 |
| $\chi_{21}$ | 3312 | 3312 | 3312 | 16 | 16 | 16 | 16 | 16 | 48 | 48 | 48 | 48 | 48 | 48 |
| $\chi_{22}$ | 3520 | 3520 | 3520 | 0 | 0 | 0 | 0 | 0 | 64 | 64 | 64 | 64 | 64 | 64 |
| $\chi_{23}$ | 5313 | 5313 | 5313 | 9 | 9 | 9 | 9 | 9 | 49 | 49 | 49 | 49 | 49 | 49 |
| $\chi_{24}$ | 5544 | 5544 | 5544 | 24 | 24 | 24 | 24 | 24 | −56 | −56 | −56 | −56 | −56 | −56 |
| $\chi_{25}$ | 5796 | 5796 | 5796 | 36 | 36 | 36 | 36 | 36 | −28 | −28 | −28 | −28 | −28 | −28 |
| $\chi_{26}$ | 10395 | 10395 | 10395 | −45 | −45 | −45 | −45 | −45 | −21 | −21 | −21 | −21 | −21 | −21 |
| $\chi_{27}$ | 276 | 20 | −12 | 12 | −12 | −4 | 4 | 0 | 36 | 4 | 20 | −12 | −4 | 4 |
| $\chi_{28}$ | 276 | 20 | −12 | −12 | 12 | 4 | −4 | 0 | 20 | −12 | 36 | 4 | −4 | 4 |
| $\chi_{29}$ | 5796 | 420 | −252 | −12 | 12 | 4 | −4 | 0 | 196 | 36 | 84 | −76 | −20 | 20 |
| $\chi_{30}$ | 5796 | 420 | −252 | 12 | −12 | −4 | 4 | 0 | 84 | −76 | 196 | 36 | −20 | 20 |
| $\chi_{31}$ | 12420 | 900 | −540 | −60 | 60 | 20 | −20 | 0 | −60 | 36 | −108 | −12 | 12 | −12 |
| $\chi_{32}$ | 12420 | 900 | −540 | 60 | −60 | −20 | 20 | 0 | −108 | −12 | −60 | 36 | 12 | −12 |
| $\chi_{33}$ | 12420 | 900 | −540 | −60 | 60 | 20 | −20 | 0 | −60 | 36 | −108 | −12 | 12 | −12 |
| $\chi_{34}$ | 12420 | 900 | −540 | 60 | −60 | −20 | 20 | 0 | −108 | −12 | −60 | 36 | 12 | −12 |
| $\chi_{35}$ | 15180 | 1100 | −660 | 60 | −60 | −20 | 20 | 0 | 300 | 76 | 92 | −132 | −28 | 28 |
| $\chi_{36}$ | 15180 | 1100 | −660 | −60 | 60 | 20 | −20 | 0 | 92 | −132 | 300 | 76 | −28 | 28 |
| $\chi_{37}$ | 27324 | 1980 | −1188 | −12 | 12 | 4 | −4 | 0 | 204 | 108 | −36 | −132 | −12 | 12 |
| $\chi_{38}$ | 27324 | 1980 | −1188 | 12 | −12 | −4 | 4 | 0 | −36 | −132 | 204 | 108 | −12 | 12 |
| $\chi_{39}$ | 42504 | 3080 | −1848 | 72 | −72 | −24 | 24 | 0 | 392 | 72 | 168 | −152 | −40 | 40 |
| $\chi_{40}$ | 42504 | 3080 | −1848 | −72 | 72 | 24 | −24 | 0 | 168 | −152 | 392 | 72 | −40 | 40 |
| $\chi_{41}$ | 57960 | 4200 | −2520 | −120 | 120 | 40 | −40 | 0 | 168 | 104 | −56 | −120 | −8 | 8 |

Table 8: The Character Table of $\bar{G} = 2^{11}{:}M_{24}$ (Continued)

| $[g]_{\bar{G}}$ | 1A | 2A | 2B | 2E | 2F | 4E | 4F | 4G | 2C | 2D | 4A | 4B | 4C | 4D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|C_{\bar{G}}(g)\|$ | 50139758 5920 | 660602880 | 389283840 | 491520 | 15360 | 32768 | 32768 | 491520 | 344064 | 6144 | 6144 | 344064 | 49152 | 49152 |
| $\chi_{42}$ | 57960 | 4200 | $-2520$ | 120 | $-120$ | $-40$ | 40 | 0 | $-56$ | $-120$ | 168 | 104 | $-8$ | 8 |
| $\chi_{43}$ | 63756 | 4620 | $-2772$ | $-108$ | 108 | 36 | $-36$ | 0 | 252 | 28 | 140 | $-84$ | $-28$ | 28 |
| $\chi_{44}$ | 63756 | 4620 | $-2772$ | 108 | $-108$ | $-36$ | 36 | 0 | 140 | $-84$ | 252 | 28 | $-28$ | 28 |
| $\chi_{45}$ | 154560 | 11200 | $-6720$ | 0 | 0 | 0 | 0 | 0 | $-448$ | 64 | $-448$ | 64 | 64 | $-64$ |
| $\chi_{46}$ | 106260 | 7700 | $-4620$ | 60 | $-60$ | $-20$ | 20 | 0 | 196 | 164 | $-140$ | $-172$ | $-4$ | 4 |
| $\chi_{47}$ | 106260 | 7700 | $-4620$ | $-60$ | 60 | 20 | $-20$ | 0 | $-140$ | $-172$ | 196 | 164 | $-4$ | 4 |
| $\chi_{48}$ | 1771 | $-21$ | 11 | 51 | 11 | 3 | $-5$ | $-1$ | 91 | $-5$ | $-21$ | 11 | 3 | $-5$ |
| $\chi_{49}$ | 1771 | $-21$ | 11 | 11 | 51 | $-5$ | 3 | $-1$ | $-21$ | 11 | 91 | $-5$ | 3 | $-5$ |
| $\chi_{50}$ | 8855 | $-105$ | 55 | 15 | 55 | $-1$ | 7 | $-5$ | 231 | 7 | $-105$ | 55 | $-1$ | $-9$ |
| $\chi_{51}$ | 8855 | $-105$ | 55 | 55 | 15 | 7 | $-1$ | $-5$ | $-105$ | 55 | 231 | 7 | $-1$ | $-9$ |
| $\chi_{52}$ | 8855 | $-105$ | 55 | $-25$ | 95 | $-9$ | 15 | $-5$ | 119 | 23 | 7 | 39 | $-1$ | $-9$ |
| $\chi_{53}$ | 8855 | $-105$ | 55 | 95 | $-25$ | 15 | $-9$ | $-5$ | 7 | 39 | 119 | 23 | $-1$ | $-9$ |
| $\chi_{54}$ | 15939 | $-189$ | 99 | $-21$ | 99 | $-5$ | 19 | $-9$ | $-77$ | 83 | 259 | 35 | $-5$ | $-13$ |
| $\chi_{55}$ | 15939 | $-189$ | 99 | 99 | $-21$ | 19 | $-5$ | $-9$ | 259 | 35 | $-77$ | 83 | $-5$ | $-13$ |
| $\chi_{56}$ | 17710 | $-210$ | 110 | $-10$ | $-90$ | 22 | 6 | $-10$ | $-98$ | 94 | 126 | 62 | $-18$ | $-2$ |
| $\chi_{57}$ | 17710 | $-210$ | 110 | $-90$ | $-10$ | 6 | 22 | $-10$ | 126 | 62 | $-98$ | 94 | $-18$ | $-2$ |
| $\chi_{58}$ | 28336 | $-336$ | 176 | 16 | 16 | 16 | 16 | $-16$ | 112 | 112 | 112 | 112 | $-16$ | $-16$ |
| $\chi_{59}$ | 10626 | $-126$ | 66 | $-54$ | $-54$ | 10 | 10 | $-6$ | $-14$ | 50 | $-14$ | 50 | $-14$ | 2 |
| $\chi_{60}$ | 10626 | $-126$ | 66 | $-54$ | $-54$ | 10 | 10 | $-6$ | $-14$ | 50 | $-14$ | 50 | $-14$ | 2 |
| $\chi_{61}$ | 21252 | $-252$ | 132 | 132 | 132 | 4 | 4 | $-12$ | 196 | 68 | 196 | 68 | 4 | $-28$ |
| $\chi_{62}$ | 31878 | $-378$ | 198 | 78 | 78 | 14 | 14 | $-18$ | 182 | 118 | 182 | 118 | $-10$ | $-26$ |
| $\chi_{63}$ | 53130 | $-630$ | 330 | $-30$ | $-30$ | 34 | 34 | $-30$ | 154 | 218 | 154 | 218 | $-38$ | $-22$ |
| $\chi_{64}$ | 31878 | $-378$ | 198 | $-66$ | $-66$ | $-2$ | $-2$ | 6 | 406 | $-42$ | $-42$ | 22 | 22 | $-26$ |
| $\chi_{65}$ | 31878 | $-378$ | 198 | $-66$ | $-66$ | $-2$ | $-2$ | 6 | $-42$ | 22 | 406 | $-42$ | 22 | $-26$ |
| $\chi_{66}$ | 79695 | $-945$ | 495 | 255 | 55 | 15 | $-25$ | $-5$ | $-273$ | 15 | 63 | $-33$ | $-9$ | 15 |
| $\chi_{67}$ | 79695 | $-945$ | 495 | 55 | 255 | $-25$ | 15 | $-5$ | 63 | $-33$ | $-273$ | 15 | $-9$ | 15 |
| $\chi_{68}$ | 79695 | $-945$ | 495 | 175 | $-105$ | 31 | $-25$ | $-5$ | 399 | $-81$ | 63 | $-33$ | 39 | $-33$ |
| $\chi_{69}$ | 79695 | $-945$ | 495 | $-105$ | 175 | $-25$ | 31 | $-5$ | 63 | $-33$ | 399 | $-81$ | 39 | $-33$ |
| $\chi_{70}$ | 127512 | $-1512$ | 792 | $-24$ | $-24$ | $-24$ | $-24$ | 24 | $-392$ | 120 | 504 | $-8$ | $-8$ | $-8$ |
| $\chi_{71}$ | 127512 | $-1512$ | 792 | $-24$ | $-24$ | $-24$ | $-24$ | 24 | 504 | $-8$ | $-392$ | 120 | $-8$ | $-8$ |
| $\chi_{72}$ | 159390 | $-1890$ | 990 | $-90$ | $-90$ | $-26$ | $-26$ | 30 | 462 | 14 | 14 | 78 | 14 | $-34$ |
| $\chi_{73}$ | 159390 | $-1890$ | 990 | $-90$ | $-90$ | $-26$ | $-26$ | 30 | 14 | 78 | 462 | 14 | 14 | $-34$ |
| $\chi_{74}$ | 159390 | $-1890$ | 990 | 230 | 150 | 6 | $-10$ | $-10$ | 462 | $-114$ | $-210$ | $-18$ | 30 | $-18$ |
| $\chi_{75}$ | 159390 | $-1890$ | 990 | 150 | 230 | $-10$ | 6 | $-10$ | $-210$ | $-18$ | 462 | $-114$ | 30 | $-18$ |
| $\chi_{76}$ | 191268 | $-2268$ | 1188 | 84 | 84 | $-44$ | $-44$ | 36 | $-252$ | 132 | $-252$ | 132 | $-60$ | 36 |
| $\chi_{77}$ | 239085 | $-2835$ | 1485 | $-315$ | 45 | $-11$ | 61 | $-15$ | 189 | $-99$ | $-147$ | $-51$ | 21 | $-3$ |
| $\chi_{78}$ | 239085 | $-2835$ | 1485 | 45 | $-315$ | 61 | $-11$ | $-15$ | $-147$ | $-51$ | 189 | $-99$ | 21 | $-3$ |
| $\chi_{79}$ | 239085 | $-2835$ | 1485 | 45 | $-75$ | 29 | 5 | $-15$ | $-147$ | $-51$ | $-483$ | $-3$ | $-27$ | 45 |
| $\chi_{80}$ | 239085 | $-2835$ | 1485 | $-75$ | 45 | 5 | 29 | $-15$ | $-483$ | $-3$ | $-147$ | $-51$ | $-27$ | 45 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 3A | 6A | 6B | 6C | 6B | 6D | 4K | 4L | 8D | 8E | 4M | 8F | 8G | 4N | 8H | 4O | 8I |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $|C_{\bar{G}}(g)|$ | 34560 | 2304 | 3456 | 5760 | 4032 | 576 | 2048 | 1024 | 256 | 2048 | 1024 | 2048 | 2048 | 768 | 256 | 256 | 768 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 5 | 5 | 5 | 5 | $-1$ | $-1$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_3$ | 0 | 0 | 0 | 0 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_4$ | 0 | 0 | 0 | 0 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_5$ | $-3$ | $-3$ | $-3$ | $-3$ | 0 | 0 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 3 | 3 | 3 | 3 |
| $\chi_6$ | $-3$ | $-3$ | $-3$ | $-3$ | 0 | 0 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 3 | 3 | 3 | 3 |
| $\chi_7$ | 9 | 9 | 9 | 9 | 0 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 0 | 0 | 0 | 0 |
| $\chi_8$ | 10 | 10 | 10 | 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_9$ | 6 | 6 | 6 | 6 | 0 | 0 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $\chi_{10}$ | 5 | 5 | 5 | 5 | $-7$ | $-7$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ |
| $\chi_{11}$ | 5 | 5 | 5 | 5 | $-7$ | $-7$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ |
| $\chi_{12}$ | 0 | 0 | 0 | 0 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | $-2$ | $-2$ | $-2$ | $-2$ |
| $\chi_{13}$ | 0 | 0 | 0 | 0 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | $-2$ | $-2$ | $-2$ | $-2$ |
| $\chi_{14}$ | 0 | 0 | 0 | 0 | 6 | 6 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 3 | 3 | 3 | 3 |
| $\chi_{15}$ | 0 | 0 | 0 | 0 | $-3$ | $-3$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_{16}$ | 0 | 0 | 0 | 0 | $-3$ | $-3$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_{17}$ | 5 | 5 | 5 | 5 | 8 | 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | $-3$ | $-3$ | $-3$ | $-3$ |
| $\chi_{18}$ | 16 | 16 | 16 | 16 | 7 | 7 | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_{19}$ | $-1$ | $-1$ | $-1$ | $-1$ | 8 | 8 | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $\chi_{20}$ | 0 | 0 | 0 | 0 | 6 | 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | $-3$ | $-3$ | $-3$ | $-3$ |
| $\chi_{21}$ | 0 | 0 | 0 | 0 | $-6$ | $-6$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{22}$ | 10 | 10 | 10 | 10 | $-8$ | $-8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{23}$ | $-15$ | $-15$ | $-15$ | $-15$ | 0 | 0 | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ |
| $\chi_{24}$ | 9 | 9 | 9 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{25}$ | $-9$ | $-9$ | $-9$ | $-9$ | 0 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 0 | 0 | 0 | 0 |
| $\chi_{26}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 3 | 3 | 3 | 3 |
| $\chi_{27}$ | 15 | $-1$ | 5 | $-3$ | 0 | 0 | 8 | 0 | 4 | $-4$ | $-4$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{28}$ | 15 | $-1$ | 5 | $-3$ | 0 | 0 | 4 | $-4$ | 8 | 0 | 0 | $-4$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{29}$ | 45 | $-3$ | 15 | $-9$ | 0 | 0 | 12 | 4 | 0 | $-8$ | $-8$ | 4 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{30}$ | 45 | $-3$ | 15 | $-9$ | 0 | 0 | 0 | $-8$ | 12 | 4 | 4 | $-8$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{31}$ | 0 | 0 | 0 | 0 | 0 | 0 | 4 | $-4$ | 8 | 0 | 0 | $-4$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{32}$ | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 | 4 | $-4$ | $-4$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{33}$ | 0 | 0 | 0 | 0 | 0 | 0 | 4 | $-4$ | 8 | 0 | 0 | $-4$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{34}$ | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 | 4 | $-4$ | $-4$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{35}$ | 15 | $-1$ | 5 | $-3$ | 0 | 0 | 4 | 12 | 0 | 8 | $-8$ | $-4$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{36}$ | 15 | $-1$ | 5 | $-3$ | 0 | 0 | 0 | 8 | 4 | 12 | $-4$ | $-8$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{37}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 4 | 12 | $-4$ | $-8$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{38}$ | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 12 | 0 | 8 | $-8$ | $-4$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{39}$ | 15 | $-1$ | 5 | $-3$ | 0 | 0 | 8 | $-8$ | 0 | $-16$ | 0 | 8 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{40}$ | 15 | $-1$ | 5 | $-3$ | 0 | 0 | 0 | $-16$ | 8 | $-8$ | 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{41}$ | 45 | $-3$ | 15 | $-9$ | 0 | 0 | $-8$ | 8 | $-16$ | 0 | 0 | 8 | 0 | 0 | 0 | 0 | 0 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 3A | 6A | 6B | 6C | 6B | 6D | 4K | 4L | 8D | 8E | 4M | 8F | 8G | 4N | 8H | 4O | 8I |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lvert C_{\bar{G}}(g)\rvert$ | 34560 | 2304 | 3456 | 5760 | 4032 | 576 | 2048 | 1024 | 256 | 2048 | 1024 | 2048 | 2048 | 768 | 256 | 256 | 768 |
| $\chi_{42}$ | 45 | −3 | 15 | −9 | 0 | 0 | −16 | 0 | −8 | 8 | 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{43}$ | −45 | 3 | −15 | 9 | 0 | 0 | −8 | 0 | −4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{44}$ | −45 | 3 | −15 | 9 | 0 | 0 | −4 | 4 | −8 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 30 | −2 | 10 | −6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{46}$ | −30 | 2 | −10 | 6 | 0 | 0 | 0 | −8 | 12 | 4 | 4 | −8 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{47}$ | −30 | 2 | −10 | 6 | 0 | 0 | 12 | 4 | 0 | −8 | −8 | 4 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{48}$ | 16 | 0 | −6 | 2 | 7 | −1 | 7 | −1 | −5 | 3 | 3 | −1 | −1 | 7 | −1 | −1 | −1 |
| $\chi_{49}$ | 16 | 0 | −6 | 2 | 7 | −1 | −5 | 3 | 7 | −1 | −1 | 3 | −1 | −1 | −1 | 7 | −1 |
| $\chi_{50}$ | 35 | 3 | −15 | 1 | −7 | 1 | 11 | 3 | −9 | −1 | 7 | −5 | −1 | −1 | −1 | −5 | 3 |
| $\chi_{51}$ | 35 | 3 | −15 | 1 | −7 | 1 | −9 | −1 | 11 | 3 | −5 | 7 | −1 | −5 | 3 | −1 | −1 |
| $\chi_{52}$ | −10 | 6 | 0 | −8 | 14 | −2 | −1 | 7 | 3 | −5 | 3 | −1 | −1 | −9 | −1 | 3 | 3 |
| $\chi_{53}$ | −10 | 6 | 0 | −8 | 14 | −2 | 3 | −5 | −1 | 7 | −1 | 3 | −1 | 3 | 3 | −9 | −1 |
| $\chi_{54}$ | 9 | 9 | −9 | −9 | 0 | 0 | −1 | −9 | 3 | 11 | −5 | 7 | −1 | 3 | −5 | 3 | 3 |
| $\chi_{55}$ | 9 | 9 | −9 | −9 | 0 | 0 | 3 | 11 | −1 | −9 | 7 | −5 | −1 | 3 | 3 | 3 | −5 |
| $\chi_{56}$ | 25 | 9 | −15 | −7 | 7 | −1 | −6 | −6 | 2 | 2 | −6 | 2 | 2 | 2 | 2 | −2 | −2 |
| $\chi_{57}$ | 25 | 9 | −15 | −7 | 7 | −1 | 2 | 2 | −6 | −6 | 2 | −6 | 2 | −2 | −2 | 2 | 2 |
| $\chi_{58}$ | −14 | 18 | −6 | −22 | −14 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{59}$ | −3 | −3 | 3 | 3 | 0 | 0 | −2 | −2 | −2 | −2 | −2 | −2 | 2 | 6 | −2 | 6 | −2 |
| $\chi_{60}$ | −3 | −3 | 3 | 3 | 0 | 0 | −2 | −2 | −2 | −2 | −2 | −2 | 2 | 6 | −2 | 6 | −2 |
| $\chi_{61}$ | −6 | −6 | 6 | 6 | 0 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | −4 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | −9 | −9 | 9 | 9 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | −2 | 6 | −2 | 6 | −2 |
| $\chi_{63}$ | −15 | −15 | 15 | 15 | 0 | 0 | −2 | −2 | −2 | −2 | −2 | −2 | 2 | −6 | 2 | −6 | 2 |
| $\chi_{64}$ | 45 | −3 | −15 | 9 | 0 | 0 | 10 | −6 | −6 | 10 | 2 | 2 | −2 | −6 | 2 | 6 | −2 |
| $\chi_{65}$ | 45 | −3 | −15 | 9 | 0 | 0 | −6 | 10 | 10 | −6 | 2 | 2 | −2 | 6 | −2 | −6 | 2 |
| $\chi_{66}$ | 0 | 0 | 0 | 0 | 21 | −3 | 7 | −1 | −5 | 3 | 3 | −1 | −1 | 7 | −1 | −1 | −1 |
| $\chi_{67}$ | 15 | 0 | 0 | 0 | 0 | 21 | −5 | 3 | 7 | −1 | −1 | 3 | −1 | −1 | −1 | 7 | −1 |
| $\chi_{68}$ | 0 | 0 | 0 | 0 | 21 | −3 | −1 | −9 | −5 | 3 | −5 | −1 | 3 | −1 | −1 | −5 | 3 |
| $\chi_{69}$ | 0 | 0 | 0 | 0 | 21 | −3 | −5 | 3 | −1 | −9 | −1 | −5 | 3 | −5 | 3 | −1 | −1 |
| $\chi_{70}$ | 45 | −3 | −15 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{71}$ | 45 | −3 | −15 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{72}$ | −45 | 3 | 15 | −9 | 0 | 0 | −6 | 10 | 10 | −6 | 2 | 2 | −2 | 6 | −2 | −6 | 2 |
| $\chi_{73}$ | −45 | 3 | 15 | −9 | 0 | 0 | 10 | −6 | −6 | 10 | 2 | 2 | −2 | −6 | 2 | 6 | −2 |
| $\chi_{74}$ | 0 | 0 | 0 | 0 | −21 | 3 | −6 | −6 | 2 | 2 | −6 | 2 | 2 | −2 | −2 | 2 | 2 |
| $\chi_{75}$ | 0 | 0 | 0 | 0 | −21 | 3 | 2 | 2 | −6 | −6 | 2 | −6 | 2 | 2 | 2 | −2 | −2 |
| $\chi_{76}$ | 0 | 0 | 0 | 0 | 0 | 0 | −4 | −4 | −4 | −4 | −4 | −4 | 4 | 0 | 0 | 0 | 0 |
| $\chi_{77}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −7 | −3 | 5 | −3 | 1 | 1 | 9 | 1 | −3 | −3 |
| $\chi_{78}$ | 0 | 0 | 0 | 0 | 0 | 0 | −3 | 5 | 1 | −7 | 1 | −3 | 1 | −3 | −3 | 9 | 1 |
| $\chi_{79}$ | 0 | 0 | 0 | 0 | 0 | 0 | −3 | 5 | 9 | 1 | 1 | 5 | −3 | −3 | 5 | −3 | −3 |
| $\chi_{80}$ | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 1 | −3 | 5 | 5 | 1 | −3 | −3 | −3 | −3 | 5 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 4H | 4I | 8A | 8B | 4J | 8C | 5A | 10A | 10B | 6E | 6F | 12A | 12B | 12C | 12D | 6G | 6H | 12E | 7A | 14A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $|C_{\bar{G}}(g)|$ | 6144 | 1024 | 6144 | 1024 | 6144 | 6144 | 2480 | 160 | 120 | 192 | 96 | 192 | 96 | 192 | 192 | 96 | 48 | 96 | 168 | 56 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ | 2 | 2 |
| $\chi_3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | $A$ | $A$ |
| $\chi_4$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | $/A$ | $/A$ |
| $\chi_5$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_6$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_7$ | 4 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_8$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | 3 | 3 | 3 | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | 1 | 1 | 1 | 1 | 1 |
| $\chi_9$ | 3 | 3 | 3 | 3 | 3 | 3 | $-2$ | $-2$ | $-2$ | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{10}$ | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| $\chi_{11}$ | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| $\chi_{12}$ | 6 | 6 | 6 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | $A$ | $A$ |
| $\chi_{13}$ | 6 | 6 | 6 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | $/A$ | $/A$ |
| $\chi_{14}$ | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | $-1$ | $-1$ |
| $\chi_{15}$ | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | $B$ | $B$ |
| $\chi_{16}$ | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | $/B$ | $/B$ |
| $\chi_{17}$ | $-7$ | $-7$ | $-7$ | $-7$ | $-7$ | $-7$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | $-2$ | $-2$ |
| $\chi_{18}$ | 3 | 3 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | $-1$ | 0 | 0 |
| $\chi_{19}$ | 8 | 8 | 8 | 8 | 8 | 8 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 0 | 0 | 0 | 1 | 1 |
| $\chi_{20}$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{21}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-3$ | $-3$ | $-3$ | 0 | 0 | 0 | 0 | 0 | 0 | $-2$ | $-2$ | $-2$ | 1 | 1 |
| $\chi_{22}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | $-2$ | 0 | 0 | 0 | $-1$ | $-1$ |
| $\chi_{23}$ | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{24}$ | $-8$ | $-8$ | $-8$ | $-8$ | $-8$ | $-8$ | $-1$ | $-1$ | $-1$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{25}$ | $-4$ | $-4$ | $-4$ | $-4$ | $-4$ | $-4$ | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{26}$ | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{27}$ | 4 | $-4$ | 4 | $-4$ | 0 | 0 | 6 | 0 | $-2$ | 3 | 1 | $-1$ | $-3$ | $-1$ | 1 | 0 | 0 | 0 | 3 | $-1$ |
| $\chi_{28}$ | $-4$ | 4 | $-4$ | 4 | 0 | 0 | 6 | 0 | $-2$ | $-1$ | $-3$ | 3 | 1 | $-1$ | 1 | 0 | 0 | 0 | 3 | $-1$ |
| $\chi_{29}$ | $-4$ | 4 | $-4$ | 4 | 0 | 0 | 6 | 0 | $-2$ | 1 | 3 | $-3$ | $-1$ | 1 | $-1$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{30}$ | 4 | $-4$ | 4 | $-4$ | 0 | 0 | 6 | 0 | $-2$ | $-3$ | $-1$ | 1 | 3 | 1 | $-1$ | 0 | 0 | 0 | 0 | 0 |
| $\chi_{31}$ | 12 | $-12$ | 12 | $-12$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $C$ | $-A$ |
| $\chi_{32}$ | $-12$ | 12 | $-12$ | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $/C$ | $-/A$ |
| $\chi_{33}$ | 12 | $-12$ | 12 | $-12$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $/C$ | $-/A$ |
| $\chi_{34}$ | $-12$ | 12 | $-12$ | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $/C$ | $-/A$ |
| $\chi_{35}$ | 4 | $-4$ | 4 | $-4$ | 0 | 0 | 0 | 0 | 0 | 3 | 1 | $-1$ | $-3$ | $-1$ | 1 | 0 | 0 | 0 | $-3$ | 1 |
| $\chi_{36}$ | $-4$ | 4 | $-4$ | 4 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-3$ | 3 | 1 | $-1$ | 1 | 0 | 0 | 0 | $-3$ | 1 |
| $\chi_{37}$ | 12 | $-12$ | 12 | $-12$ | 0 | 0 | $-6$ | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | $-1$ |
| $\chi_{38}$ | $-12$ | 12 | $-12$ | 12 | 0 | 0 | $-6$ | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | $-1$ |
| $\chi_{39}$ | 8 | $-8$ | 8 | $-8$ | 0 | 0 | $-6$ | 0 | 2 | $-1$ | $-3$ | 3 | 1 | $-1$ | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{40}$ | $-8$ | 8 | $-8$ | 8 | 0 | 0 | $-6$ | 0 | 2 | 3 | 1 | $-1$ | $-3$ | $-1$ | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{41}$ | $-8$ | 8 | $-8$ | 8 | 0 | 0 | 0 | 0 | 0 | $-3$ | $-1$ | 1 | 3 | 1 | $-1$ | 0 | 0 | 0 | 0 | 0 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 4H | 4I | 8A | 8B | 4J | 8C | 5A | 10A | 10B | 6E | 6F | 12A | 12B | 12C | 12D | 6G | 6H | 12E | 7A | 14A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|C_{\bar{G}}(g)\|$ | 6144 | 1024 | 6144 | 1024 | 6144 | 6144 | 2480 | 160 | 120 | 192 | 96 | 192 | 96 | 192 | 192 | 96 | 48 | 96 | 168 | 56 |
| $\chi_{42}$ | 8 | −8 | 8 | −8 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | −3 | −1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{43}$ | −4 | 4 | −4 | 4 | 0 | 0 | 6 | 0 | −2 | 3 | 1 | −1 | −3 | −1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{44}$ | 4 | −4 | 4 | −4 | 0 | 0 | 6 | 0 | −2 | −1 | −3 | 3 | 1 | −1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | −2 | 2 | −2 | −2 | 2 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{46}$ | −12 | 12 | −12 | 12 | 0 | 0 | 0 | 0 | 0 | −2 | 2 | −2 | 2 | 2 | −2 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{47}$ | 12 | −12 | 12 | −12 | 0 | 0 | 0 | 0 | 0 | −2 | 2 | −2 | 2 | 2 | −2 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{48}$ | 11 | 3 | −5 | 3 | −1 | −1 | 1 | −1 | 1 | 4 | −2 | 0 | 2 | 0 | −2 | 3 | −1 | −1 | 0 | 0 |
| $\chi_{49}$ | 3 | −5 | 3 | 11 | −1 | −1 | 1 | −1 | 1 | 0 | 2 | 4 | −2 | 0 | −2 | −1 | 3 | −1 | 0 | 0 |
| $\chi_{50}$ | −1 | 7 | 7 | −1 | 3 | −5 | 0 | 0 | 0 | 3 | 1 | 3 | 1 | −1 | −3 | −3 | 1 | 1 | 0 | 0 |
| $\chi_{51}$ | −1 | 7 | 7 | −1 | −5 | 3 | 0 | 0 | 0 | 3 | 1 | 3 | 1 | −1 | −3 | 1 | −3 | 1 | 0 | 0 |
| $\chi_{52}$ | −9 | −1 | 15 | 7 | 3 | −5 | 0 | 0 | 0 | 2 | −4 | −2 | 0 | 2 | 0 | 2 | 2 | −2 | 0 | 0 |
| $\chi_{53}$ | 7 | 15 | −1 | −9 | −5 | 3 | 0 | 0 | 0 | −2 | 0 | 2 | −4 | 2 | 0 | 2 | 2 | −2 | 0 | 0 |
| $\chi_{54}$ | −5 | −13 | 11 | 19 | −1 | −1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{55}$ | 19 | 11 | −13 | −5 | −1 | −1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{56}$ | −2 | −2 | −10 | −10 | −2 | 6 | 0 | 0 | 0 | 1 | 1 | −3 | 5 | −3 | 1 | −1 | 3 | −1 | 0 | 0 |
| $\chi_{57}$ | −10 | −10 | −2 | −2 | 6 | −2 | 0 | 0 | 0 | −3 | 5 | 1 | 1 | −3 | 1 | 3 | −1 | −1 | 0 | 0 |
| $\chi_{58}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 1 | −2 | −2 | −2 | −2 | 2 | 2 | −2 | −2 | 2 | 0 | 0 |
| $\chi_{59}$ | 2 | −14 | −14 | 2 | 2 | 2 | 1 | −1 | 1 | 1 | −1 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{60}$ | 2 | −14 | −14 | 2 | 2 | 2 | 1 | −1 | 1 | 1 | −1 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{61}$ | 12 | 12 | 12 | 12 | −4 | −4 | 2 | −2 | 2 | −2 | 2 | −2 | 2 | −2 | 2 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | 14 | −2 | −2 | 14 | −2 | −2 | 2 | −2 | 2 | −1 | 1 | −1 | 1 | −1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{63}$ | −14 | 2 | 2 | −14 | 2 | 2 | 0 | 0 | 0 | 1 | −1 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{64}$ | −10 | −10 | −2 | −2 | −2 | 6 | 3 | −3 | 3 | 1 | −3 | −3 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{65}$ | −2 | −2 | −10 | −10 | 6 | −2 | 3 | −3 | 3 | −3 | 1 | 1 | −3 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{66}$ | −33 | −9 | 15 | −9 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −3 | 1 | 1 | 0 | 0 |
| $\chi_{67}$ | −9 | 15 | −9 | −33 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −3 | 1 | 0 | 0 |
| $\chi_{68}$ | 15 | −9 | −9 | 15 | 3 | −5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −3 | 1 | 0 | 0 |
| $\chi_{69}$ | 15 | −9 | −9 | 15 | −5 | 3 | 0 | 0 | 0 | 1 | −3 | −3 | 1 | 1 | 0 | −3 | 1 | 1 | 0 | 0 |
| $\chi_{70}$ | 8 | 8 | −8 | −8 | 8 | −8 | −3 | 3 | −3 | 1 | −3 | −3 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{71}$ | −8 | −8 | 8 | 8 | −8 | 8 | −3 | 3 | −3 | −3 | 1 | 1 | −3 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{72}$ | −10 | −10 | −2 | −2 | −2 | 6 | 0 | 0 | 0 | 3 | −1 | −1 | 3 | −1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{73}$ | −2 | −2 | −10 | −10 | 6 | −2 | 0 | 0 | 0 | −1 | 3 | 3 | −1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{74}$ | 6 | 6 | −18 | −18 | 6 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 3 | −1 | 0 | 0 |
| $\chi_{75}$ | −18 | −18 | 6 | 6 | −2 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | −1 | −1 | 0 | 0 |
| $\chi_{76}$ | 12 | 12 | 12 | 12 | −4 | −4 | 3 | −3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{77}$ | −3 | 21 | 21 | −3 | 1 | −7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{78}$ | −3 | 21 | 21 | −3 | −7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{79}$ | −3 | −27 | −3 | 21 | 9 | −7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{80}$ | 21 | −3 | −27 | −3 | −7 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 7B | 14B | 8J | 8K | 16A | 16B | 10C | 10D | 20A | 11A | 22A | 12F | 24A | 12G | 24B | 12H | 12I | 14C | 28A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $|C_{\bar{G}}(g)|$ | 168 | 56 | 256 | 128 | 32 | 256 | 80 | 40 | 80 | 22 | 22 | 48 | 48 | 48 | 48 | 24 | 24 | 28 | 28 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 2 | 2 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 | 0 |
| $\chi_3$ | /A | /A | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | -A | -A |
| $\chi_4$ | A | A | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | -/A | -/A |
| $\chi_5$ | 0 | 0 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 0 | 0 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_6$ | 0 | 0 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 0 | 0 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_7$ | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | -1 | -1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_8$ | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | -1 | -1 |
| $\chi_9$ | 0 | 0 | -1 | -1 | -1 | -1 | -2 | -2 | -2 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{10}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | -1 | 1 | 1 | 0 | 0 |
| $\chi_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | -1 | 1 | 1 | 0 | 0 |
| $\chi_{12}$ | /A | /A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | A | A |
| $\chi_{13}$ | A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | /A | /A |
| $\chi_{14}$ | -1 | -1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 |
| $\chi_{15}$ | /B | /B | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| $\chi_{16}$ | B | B | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| $\chi_{17}$ | -2 | -2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{18}$ | 0 | 0 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| $\chi_{19}$ | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 0 | 0 | -1 | -1 | -1 | -1 | 0 | 0 | 1 | 1 |
| $\chi_{20}$ | 2 | 2 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{21}$ | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 |
| $\chi_{22}$ | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $\chi_{23}$ | 0 | 0 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{24}$ | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{25}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{26}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{27}$ | 3 | -1 | 2 | -2 | 0 | 0 | 2 | -2 | 0 | 1 | -1 | 1 | -1 | -1 | 1 | 0 | 0 | 1 | -1 |
| $\chi_{28}$ | 3 | -1 | 0 | 0 | 2 | -2 | -2 | 2 | 0 | 1 | -1 | -1 | 1 | 1 | -1 | 0 | 0 | -1 | 1 |
| $\chi_{29}$ | 0 | 0 | 0 | 0 | -2 | 2 | -2 | 2 | 0 | -1 | 1 | -1 | 1 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{30}$ | 0 | 0 | -2 | 2 | 0 | 0 | 2 | -2 | 0 | -1 | 1 | 1 | -1 | -1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{31}$ | /C | -/A | 0 | 0 | -2 | 2 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | A | -A |
| $\chi_{32}$ | C | -A | 0 | 0 | -2 | 2 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -A | A |
| $\chi_{33}$ | C | -A | -2 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | /A | -/A |
| $\chi_{34}$ | C | -A | -2 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -/A | /A |
| $\chi_{35}$ | -3 | 1 | 0 | 0 | 2 | -2 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | -1 | 1 | 0 | 0 | -1 | 1 |
| $\chi_{36}$ | -3 | 1 | 2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 1 | -1 | 0 | 0 | 1 | -1 |
| $\chi_{37}$ | 3 | -1 | -2 | 2 | 0 | 0 | -2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 |
| $\chi_{38}$ | 3 | -1 | 0 | 0 | -2 | 2 | 2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 |
| $\chi_{39}$ | 0 | 0 | 0 | 0 | 0 | 0 | 2 | -2 | 0 | 0 | 0 | -1 | 1 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{40}$ | 0 | 0 | 0 | 0 | 0 | 0 | -2 | 2 | 0 | 0 | 0 | 1 | -1 | -1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{41}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 1 | -1 | -1 | 1 | 0 | 0 | 0 | 0 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 7B | 14B | 8J | 8K | 16A | 16B | 10C | 10D | 20A | 11A | 22A | 12F | 24A | 12G | 24B | 12H | 12I | 14C | 28A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $|C_{\bar{G}}(g)|$ | 168 | 56 | 256 | 128 | 32 | 256 | 80 | 40 | 80 | 22 | 22 | 48 | 48 | 48 | 48 | 24 | 24 | 28 | 28 |
| $\chi_{42}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | −1 | 1 | 1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{43}$ | 0 | 0 | −2 | 2 | 0 | 0 | 2 | −2 | 0 | 0 | 0 | −1 | 1 | 1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{44}$ | 0 | 0 | 0 | 0 | −2 | 2 | −2 | 2 | 0 | 0 | 0 | 1 | −1 | −1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{46}$ | 0 | 0 | 2 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{47}$ | 0 | 0 | 0 | 0 | 2 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{48}$ | 0 | 0 | 1 | 1 | −1 | −1 | 1 | 1 | −1 | 0 | 0 | 2 | 0 | 0 | −2 | 1 | −1 | 0 | 0 |
| $\chi_{49}$ | 0 | 0 | −1 | −1 | 1 | 1 | 1 | 1 | −1 | 0 | 0 | 0 | 2 | −2 | 0 | −1 | 1 | 0 | 0 |
| $\chi_{50}$ | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | −1 | −1 | 1 | 1 | −1 | 1 | 0 | 0 |
| $\chi_{51}$ | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | −1 | −1 | 1 | 1 | 1 | −1 | 0 | 0 |
| $\chi_{52}$ | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | −2 | 2 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{53}$ | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | −2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| $\chi_{54}$ | 0 | 0 | 1 | 1 | −1 | −1 | −1 | −1 | 1 | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{55}$ | 0 | 0 | −1 | −1 | 1 | 1 | −1 | −1 | 1 | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{56}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 1 | −1 | −1 | 1 | 0 | 0 |
| $\chi_{57}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 1 | −1 | 1 | 1 | −1 | 0 | 0 |
| $\chi_{58}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{59}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | −1 | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{60}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | −1 | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{61}$ | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | 0 | 0 | 0 | 0 | 0 | 0 | −2 | −2 | 2 | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{63}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{64}$ | 0 | 0 | 0 | 0 | 0 | 0 | −1 | −1 | 1 | 0 | 0 | −1 | 1 | −1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{65}$ | 0 | 0 | 0 | 0 | 0 | 0 | −1 | −1 | 1 | 0 | 0 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{66}$ | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 0 | 0 |
| $\chi_{67}$ | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 1 | 0 | 0 |
| $\chi_{68}$ | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 1 | 0 | 0 |
| $\chi_{69}$ | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 0 | 0 |
| $\chi_{70}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | −1 | 0 | 0 | −1 | 1 | −1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{71}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | −1 | 0 | 0 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{72}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 1 | −1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{73}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 1 | −1 | 0 | 0 | 0 | 0 |
| $\chi_{74}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 0 | 0 |
| $\chi_{75}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 1 | 0 | 0 |
| $\chi_{76}$ | 0 | 0 | 0 | 0 | 0 | 0 | −1 | −1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{77}$ | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{78}$ | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{79}$ | 0 | 0 | 1 | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{80}$ | 0 | 0 | −1 | −1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

<div align="center">Table 8: *(Continued)*</div>

| $[g]_{\bar{G}}$ | 14D | 28B | 15A | 30A | 15B | 30B | 21A | 21B | 23A | 23B |
|---|---|---|---|---|---|---|---|---|---|---|
| $\|C_{\bar{G}}(g)\|$ | 28 | 28 | 30 | 30 | 30 | 30 | 21 | 21 | 23 | 23 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | 0 | 0 |
| $\chi_3$ | $-/A$ | $-/A$ | 0 | 0 | 0 | 0 | $A$ | $/A$ | $-1$ | $-1$ |
| $\chi_4$ | $-A$ | $-A$ | 0 | 0 | 0 | 0 | $/A$ | $A$ | $-1$ | $-1$ |
| $\chi_5$ | 0 | 0 | $D$ | $D$ | $/D$ | $/D$ | 0 | 0 | 1 | 1 |
| $\chi_6$ | 0 | 0 | $/D$ | $/D$ | $D$ | $D$ | 0 | 0 | 1 | 1 |
| $\chi_7$ | 0 | 0 | $-1$ | $-1$ | $-1$ | $-1$ | 0 | 0 | $-1$ | $-1$ |
| $\chi_8$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $\chi_9$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{10}$ | 0 | 0 | 0 | 0 | $0-$ | 0 | 0 | 0 | $E$ | $/E$ |
| $\chi_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $/E$ | $E$ |
| $\chi_{12}$ | $/A$ | $/A$ | 0 | 0 | 0 | 0 | $A$ | $/A$ | 1 | 1 |
| $\chi_{13}$ | $A$ | $A$ | 0 | 0 | 0 | 0 | $/A$ | $A$ | 1 | 1 |
| $\chi_{14}$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 | $-1$ | $-1$ | 0 | 0 |
| $\chi_{15}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-A$ | $-/A$ | 0 | 0 |
| $\chi_{16}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-/A$ | $-A$ | 0 | 0 |
| $\chi_{17}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $\chi_{18}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{19}$ | 1 | 1 | $-1$ | $-1$ | $-1$ | $-1$ | 1 | 1 | 0 | 0 |
| $\chi_{20}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | 0 | 0 |
| $\chi_{21}$ | $-1$ | $-1$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $\chi_{22}$ | 1 | 1 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | 1 | 1 |
| $\chi_{23}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{24}$ | 0 | 0 | $-1$ | $-1$ | $-1$ | $-1$ | 0 | 0 | 1 | 1 |
| $\chi_{25}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{26}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ |
| $\chi_{27}$ | 1 | $-1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{28}$ | $-1$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{29}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{30}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{31}$ | $/A$ | $-/A$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{32}$ | $-/A$ | $/A$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{33}$ | $A$ | $-A$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{34}$ | $-A$ | $A$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{35}$ | $-1$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{36}$ | 1 | $-1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{37}$ | 1 | $-1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{38}$ | $-1$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{39}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{40}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{41}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{42}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 8: *(Continued)*

| $[g]_{\bar{G}}$ | 14D | 28B | 15A | 30A | 15B | 30B | 21A | 21B | 23A | 23B |
|---|---|---|---|---|---|---|---|---|---|---|
| $|C_{\bar{G}}(g)|$ | 28 | 28 | 30 | 30 | 30 | 30 | 21 | 21 | 23 | 23 |
| $\chi_{43}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{44}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{46}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{47}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{48}$ | 0 | 0 | 1 | -1 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{49}$ | 0 | 0 | 1 | -1 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{50}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{51}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{52}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{53}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{54}$ | 0 | 0 | -1 | 1 | -1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{55}$ | 0 | 0 | -1 | 1 | -1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{56}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{57}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{58}$ | 0 | 0 | 1 | -1 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{59}$ | 0 | 0 | $D$ | $-D$ | $/D$ | $-/D$ | 0 | 0 | 0 | 0 |
| $\chi_{60}$ | 0 | 0 | $/D$ | $-/D$ | $D$ | $-D$ | 0 | 0 | 0 | 0 |
| $\chi_{61}$ | 0 | 0 | -1 | 1 | -1 | 1 | 0 | 0 | 0 | 0 |
| $\chi_{62}$ | 0 | 0 | 1 | -1 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\chi_{63}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{64}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{65}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{66}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{67}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{68}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{69}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{70}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{71}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{72}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{73}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{74}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{75}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{76}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{77}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{78}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{79}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\chi_{80}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

In Table 8, we take $A = \dfrac{-1+\sqrt{-7}}{2}, B = -1 + \sqrt{-7}, C = \dfrac{-3+3\sqrt{-7}}{2}, D = \dfrac{-1+\sqrt{-15}}{2}$ and $E = \dfrac{-1+\sqrt{-23}}{2}$ where $/A, /B, /C /D$ and $/E$ are conjugates of $A$, $B$, $C$ $D$ and $E$ respectively.

Table 9: Fusion of $2^{11}{:}M_{24}$ into $Co_1$

| $[x]_{\overline{G}}$ | $\rightarrow$ | $[y]_{CO_1}$ | $[x]_{\overline{G}}$ | $\rightarrow$ | $[y]_{CO_1}$ | $[x]_{\overline{G}}$ | $\rightarrow$ | $[y]_{CO_1}$ | $[x]_{\overline{G}}$ | $\rightarrow$ | $[y]_{CO_1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1A | | 1A | 4L | | 4D | 8F | | 8D | 14B | | 14B |
| 2A | | 2A | 4M | | 4F | 8G | | 8D | 14C | | 14B |
| 2B | | 2c | 4N | | 4E | 8H | | 8F | 14D | | 14B |
| 2C | | 2A | 4O | | 4F | 8I | | 8E | 15A | | 15D |
| 2D | | 2C | 5A | | 5B | 8J | | 8E | 15B | | 15D |
| 2E | | 2C | 6A | | 6E | 8K | | 8F | 16B | | 16A |
| 2F | | 2B | 6B | | 6C | 10A | | 10D | 16B | | 16B |
| 3A | | 3B | 6C | | 6G | 10B | | 10F | 11A | | 11A |
| 3B | | 3D | 6D | | 6I | 10C | | 10C | 20A | | 20B |
| 4A | | 4A | 6E | | 6E | 10D | | 10F | 21A | | 21C |
| 4B | | 4B | 6F | | 6G | 12A | | 12E | 21B | | 21C |
| 4C | | 4C | 6G | | 6H | 12B | | 12G | 22A | | 22A |
| 4D | | 4D | 6H | | 6I | 12C | | 12G | 23A | | 23A |
| 4E | | 4B | 7A | | 7B | 12D | | 12G | 23B | | 23B |
| 4F | | 4D | 7B | | 7B | 12E | | 12I | 24A | | 24C |
| 4G | | 4F | 8A | | 8A | 12F | | 12J | 24B | | 24E |
| 4H | | 4B | 8B | | 8B | 12G | | 12M | 28A | | 28A |
| 4I | | 4D | 8C | | 8B | 12H | | 12L | 28B | | 28A |
| 4J | | 4F | 8D | | 8C | 12I | | 12M | 30A | | 30D |
| 4K | | 4C | 8E | | 8D | 14A | | 14B | 30B | | 30D |

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