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Original Article

On a group of the form $2^{11}{:}M_{24}$

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ABSTRACT: The Conway group Co_1 is one of the 26 sporadic simple groups. It is the largest of the three Conway groups with order $4157776806543360000 = 2^{21}.3^9.5^4.7^2.11.13.23$ and has 22 conjugacy classes of maximal subgroups. In this paper, we discuss a group of the form $\overline{G} = N{:}G$, where $N = 2^{11}$ and $G = M_{24}$. This group $\overline{G} = N{:}G = 2^{11}{:}M_{24}$ is a split extension of an elementary abelian group $N = 2^{11}$ by a Mathieu group $G = M_{24}$. Using the computed Fischer matrices for each class representative g of G and ordinary character tables of the inertia factor groups of G, we obtain the full character table of $\overline{G}$. The complete fusion of $\overline{G}$ into its mother group Co_1 is also determined using the permutation character of Co_1.

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1. Introduction

The group $\overline{G}$ has order 501397585920 and $[Co_1 : \overline{G}] = 8292375$. In this paper, we will compute the conjugacy classes, inertia factor groups and Fischer-Cliffford matrices which lead to the construction of the ordinary character table of $\overline{G} = N{:}G = 2^{11}{:}M_{24}$ using the technique of coset analysis and the theory of Fischer-Clifford matrices. These methods have been largely studied and applied by many researchers in their journals, PhD and Masters Dissertations (see for example: [1, 2, 5, 3, 4, 7, 10, 9, 13, 14]).
The fusion of the classes of $\overline{G}$ into the classes of its mother group Co_1 will be completely obtained by the permutation character of Co_1 on $\overline{G}$.

To construct $\overline{G}$ inside its cover Co_1, we first need to construct Co_1 as follows:
From the online Atlas V_3 [19], we obtain two 24×24 matrix generators a and b of Co_1 over GF(2) with $o(a) = 2$ and $o(b) = 3$. Then with the help of GAP [12], $Co_1 = \langle\{a, b\}\rangle$. Again from online Atlas V_3 [19], We then extract **programme A** (see Section 7) and implement it in GAP [12] to obtain two 24×24 matrix generators c and d of

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$\overline{G}$ over GF(2) with $o(c) = 2$ and $o(d) = 6$ presented below.

$$c = \begin{pmatrix}
1&0&1&0&0&1&1&1&1&1&0&1&1&0&0&1&0&0&0&1&0&0&0&0\\
0&1&0&1&0&0&1&0&1&0&1&1&0&1&0&1&1&0&1&1&1&1&0&0\\
1&1&0&1&1&1&0&1&0&0&0&1&1&0&1&1&0&0&1&1&1&0&1&0\\
1&1&0&0&0&0&0&0&1&0&1&0&0&1&0&0&0&1&0&0&1&1&0&1\\
1&0&1&0&1&0&1&0&0&1&0&0&1&0&1&1&0&1&0&0&1&1&1&0\\
1&1&1&1&1&0&1&1&0&1&0&1&1&0&0&0&0&1&0&0&1&1&0&0\\
0&1&0&0&0&1&0&1&0&1&0&1&1&0&1&1&1&0&0&1&1&1&1&1\\
0&0&0&0&0&0&1&0&1&0&0&0&1&0&1&0&0&0&0&0&1&0&0&1\\
1&1&0&0&1&1&1&1&1&1&1&0&0&0&1&0&0&1&0&1&0&1&1&1\\
1&1&1&1&0&0&0&1&0&1&1&0&0&1&0&0&1&0&1&0&0&0&0&1\\
1&1&0&0&0&0&0&1&0&0&0&1&0&0&1&0&0&0&0&1&1&0&0&0\\
1&1&0&0&0&1&0&0&1&1&0&1&1&1&0&1&0&1&0&1&0&1&1&1\\
0&1&1&1&0&1&0&0&1&0&1&1&1&1&1&0&1&1&1&1&1&1&1&1\\
0&1&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1&0&1&1&0&1&1&1\\
0&0&1&1&0&0&1&1&0&0&0&1&1&0&1&1&1&1&1&0&1&0&1&0\\
1&1&1&0&1&0&0&0&1&1&1&0&0&1&1&1&0&0&1&1&0&1&0&1\\
1&1&0&1&1&1&1&1&1&1&0&0&1&0&0&1&0&1&1&1&0&1&1&1\\
1&1&1&0&0&0&1&1&1&0&1&1&1&1&0&1&1&1&1&0&0&1&1&1\\
1&0&1&0&1&1&1&0&0&1&1&1&0&1&0&0&1&1&1&1&0&1&0&1\\
0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&1&1&0&0&1&1&0&1&1\\
0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&1&1&0&0&0&0&0&1&1\\
1&0&0&0&0&0&1&1&1&0&1&1&1&0&1&0&0&1&1&0&1&1&1&0\\
1&0&0&0&0&0&1&1&1&0&1&1&1&0&1&0&0&1&1&1&0&0&0&0\\
1&1&1&0&1&1&0&0&0&1&0&1&0&1&0&1&0&1&0&1&1&1&1&0
\end{pmatrix}
d = \begin{pmatrix}
1&1&1&1&0&0&1&0&1&1&1&1&1&1&1&1&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&1&1&0&1&1&1&1&1&1&1&1&1&0&1&1&1\\
0&1&0&0&1&0&1&1&1&1&1&0&1&1&0&1&0&0&0&0&1&1&1&1\\
0&1&1&1&0&1&0&0&0&1&1&1&0&1&1&1&0&1&1&1&1&0&0&0\\
1&1&0&1&1&1&1&0&0&1&0&0&1&0&1&1&0&1&0&0&0&1&1&0\\
0&0&1&1&0&0&1&1&1&1&1&0&0&1&1&0&1&0&0&0&1&1&0&0\\
0&1&1&0&0&1&1&1&0&0&0&0&0&0&0&1&0&0&1&1&0&0&1&1\\
1&0&0&0&0&0&1&1&0&0&0&0&1&1&1&1&1&1&1&0&0&1&0&0\\
1&1&0&1&1&1&1&1&0&0&0&0&1&0&0&1&1&0&0&0&0&0&1&1\\
1&0&0&0&0&1&1&1&1&1&1&1&0&0&1&0&1&1&1&1&1&1&1&1\\
0&1&1&1&0&1&0&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&1&0\\
1&1&0&0&1&1&0&1&0&1&1&0&0&0&0&0&1&1&0&0&1&0&1&0\\
0&1&0&1&0&1&1&0&1&1&1&1&1&1&0&1&1&1&1&1&1&0&1&0\\
1&0&1&0&1&1&0&0&0&1&0&1&0&1&0&0&1&1&0&0&0&0&1&1\\
0&0&1&0&0&0&1&0&0&1&0&1&1&0&0&0&1&0&1&0&0&1&0&1\\
0&1&1&0&1&1&0&0&0&0&1&1&1&1&1&1&0&1&0&0&1&1&1&1\\
1&0&1&0&0&1&1&1&1&1&1&1&0&0&1&0&1&0&0&0&1&1&0&0\\
0&1&0&0&1&1&0&1&0&1&1&0&1&1&0&1&0&1&1&1&1&1&1&1\\
1&1&1&0&0&1&1&0&0&0&0&0&0&1&0&1&0&0&0&0&1&1&0&0\\
0&0&0&1&0&1&1&0&1&1&1&1&1&1&1&0&1&1&1&0&1&0&1&1\\
0&0&0&0&0&1&0&1&1&1&1&1&0&0&1&0&0&0&1&0&0&1&1&0\\
0&0&1&0&1&0&1&1&0&0&0&1&1&0&1&0&0&1&0&0&1&0&0&1\\
0&1&1&1&0&0&0&1&1&1&1&0&0&1&0&0&0&1&1&0&0&1&1&0\\
0&1&0&0&1&1&1&1&0&1&0&1&0&1&1&1&1&1&1&1&0&1&1&0
\end{pmatrix}.$$

Thus, we have that $\overline{G} = 2^{11}{:}M_{24}=\langle$c, d$\rangle$. After constructing $\overline{G}$ in GAP, we observe that it has two non-trivial normal subgroups and only one of these is of order 2048 and is an elementary abelian 2-group with 11 generators n_1, n_2,n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10} and n_{11} each of order 2. This group is our $\{N\}$ and $\{N\} = 2^{11}$. Therefore, up to isomorphism, there is only one group of the type $2^{11}{:}\, M_{24}$ inside Co_1.

The complement of $N = 2^{11}$ in $\overline{G} = 2^{11}{:}\, M_{24}$ is $G = M_{24}$ generated by two 24×24 matrices each of order 2 which are then reduced to two 11×11 matrices g_1 and g_2 using **Programmme F** (see section 7).

$$g_1 = \begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&1&0&0&0&0&1&1&1
\end{pmatrix}
g_2 = \begin{pmatrix}
0&0&1&0&0&0&0&0&0&0&0\\
0&1&1&0&0&0&0&0&0&0&0\\
1&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0\\
1&0&1&1&1&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&0
\end{pmatrix}
\text{ with } o(g_1) = 2 \text{ and } o(g_2) = 3$$

2. The Action of $G = M_{24}$ on 2^{11} and $Irr(2^{11})$

$\bar{G} = 2^{11}{:}M_{24}$ is a split extension of $N = 2^{11}$ by $G = M_{24}$, where N is considered as vector space of dimension 11 over $GF(2)$. Using GAP, the action of $G = M_{24}$ on $N = 2^{11}$ results into 3 orbits of lengths 1, 759 and 1288 with corresponding point stabilizers M_{24}, $2^4{:}A_8$ and $M_{12}{:}2$ of orders 244823040, 322560 and 190080 respectively. Since G has 3 orbits from its action on N, it must have 3 orbits from its action on $Irr(2^{11})$ by Brauer's theorem in [18, Theorem 2.6.7]. This is exactly the case here, we have 3 orbits from the action on the conjugacy classes and the irreducible characters. That is, we have 1, 759 and 1288 orbit lengths from the action on the conjugacy classes and 1, 276 and 1771 on the irreducible characters.

3. Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$

In this section, the technique of coset analysis is used to compute the conjugacy classes of the elements of $\overline{G}$. This technique was developed and first used by Moori [18] and since then, it has been applied by many other researchers to calculate the conjugacy classes of groups of extension type. We give a brief summary of coset analysis. We look at the action of $\bar{G}$ on $N\bar{g}$, for the split extension it suffices to look at Ng, $g \in G$. First N acts on Ng and breaks it into k orbits say $Q_1, Q_2, \ldots, Q_k$. Then we act $C_G(g)$ on these orbits and f_j of them fuse to form one orbit $\triangle_j$ with $\sum_j f_j = k$. For this we use **Programme B** and **Programme C** for the power maps in [9]. The reader is referred to Basheer [3], Chikopela [6], Chileshe [9] and Kapata [13] among others for further details and recent application of this technique. Now, corresponding to 26 conjugacy classes of M_{24}, we obtain 80 conjugacy classes of $\overline{G}$ which are listed in Table 1. In this table, the $m_j's$ are the weights which will play a big role in computation of Clifford-Fischer matrices later in section 3 and are calculated by $m_j = \frac{f_j \times |N|}{k}$.

Theorem 3.1. *Let G be a group, p be the number of conjugacy classes of G and $Irr(G) = \{\chi_1, \chi_2, \cdots, \chi_q\}$ be the set of irreducible characters of G, then $p = q$.*

Proof. (See Chileshe [8]). □

4. Fischer-Clifford Matrices of $\bar{G}$

After obtaining the conjugacy classes of $\overline{G}$ and the fusion maps of the inertia factor groups H_2 and H_3 into G, the Fischer-Clifford matrices of the group $\bar{G} = 2^{11}{:}M_{24}$ will now be computed. **Programme D** is extensively used for the automatic determination of a possible candidate for each Fischer-Clifford matrix $M(g)$, $g \in G$, of $\overline{G}$. The properties of Fischer-Clifford matrices discussed in [3], [10], [16], and [17] are used to rearrange the rows and columns in order to get the unique matrix $M(g)$ for $\bar{G} = 2^{11}{:}M_{24}$. Note that **Programme D** only applies on split extensions $N{:}G$, where N is elementary abelian. Since $N = 2^{11}$ is an elementary abelian p-group, then all the following properties of the Fischer matrix $M(g)$ that help with the computation of its entries will hold:

(1) $a_j^{(1,g)} = 1$ for all $j \in \{1,\ 2, \ldots, c(g)\}$,

(2) $|X(g)| = |R(g)|$,

(3) $\displaystyle\sum_{(i,y)\in R(g)} \frac{|C_{H_i}(y)| a^j_{(i,y)} \overline{a^{j'}_{(i,y)}}}{|C_{\bar{G}}(x_j)|} = \delta_{jj'}$, called the column orthogonality relation,

(4) $a_1^{(i,y)} = \dfrac{|C_G(g)|}{|C_{H_i}(y)|}$,

(5) $|a_1^{(i,y)}| \geq |a_j^{(i,y)}|$.

For example, let us consider the conjugacy class $3A$ of $G = M_{24}$, then applying **property (e)** of **Theorem 5.2.4** in [15], $M(3A)$ takes the following form with corresponding weights attached to the rows and columns,

$$\begin{array}{rcl|cccc} & & & |C_{\overline{G}}(3A)| & |C_{\overline{G}}(6A)| & |C_{\overline{G}}(6B)| & |C_{\overline{G}}(6C)| \\ & & & 34560 & 2304 & 3456 & 5760 \\ |C_{H_1}(3A)| & = & 1080 & 1 & 1 & 1 & 1 \\ |C_{H_2}(3a)| & = & 72 & 15 & a & b & c \\ |C_{H_3}(3a)| & = & 1080 & 1 & d & e & f \\ |C_{H_3}(3b)| & = & 72 & 15 & g & h & i \\ & & m_{ij} & 64 & 960 & 640 & 384 \end{array}$$

In order to determine the entries a, b, c, d, e, f, g, h and i of the Fischer-Clifford matrix $M(3A)$, the GAP output for programme D is first generated and arranged using properties of Fischer-Clifford matrices to obtain:

$$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 15 & -1 & 3 & -5 \\ 1 & 1 & -1 & -1 \\ 15 & -1 & -3 & 5 \end{pmatrix}$$

Similarly,

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 276 & 20 & -12 \\ 1771 & -21 & 11 \end{pmatrix}$$

$$M(21A) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(21B) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(23A) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(23B) = \begin{pmatrix} 1 \end{pmatrix}$$

Proceeding this way on every conjugacy class of G, we obtain all the 26 corresponding Fischer-Clifford matrices of $\bar{G}$. **Programme G** computes the Fischer Matrix of $\bar{G}$ on $1_G \in G$. We list all the Fischer matrices in Table 7.

5. Character Table of $\bar{G}$

By Theorem 3.1, the character table is easily seen as a square matrix with columns indexed by the conjugacy classes and rows corresponding to the irreducible characters of G.

Definition 5.1 ([9]). *Let G be a group and $\{g_1, g_2, \cdots, g_l\}$ be the conjugacy class representatives of G. Let $Irr(G) = \{\chi_1, \chi_2, \cdots, \chi_l\}$ be a set of irreducible characters of G. Then the* ***character table*** *of G is an $l \times l$ matrix whose entries are the values $\chi_i(g_j)$ for $i, j \in \{1, 2, \cdots, l\}$. The columns of this matrix are indexed by g_j and the rows by χ_i for $i, j \in \{1, 2, \cdots, l\}$.*

Now, once we have obtained the conjugacy classes of $\bar{G} = 2^{11}{:}M_{24}$, the character tables of all the inertia factor groups H_1, H_2 and H_3 presented in Tables 4, 5 and 6 respectively, the fusions of conjugacy classes of the inertia factor groups H_1 and H_2 into classes of $G = M_{24}$ given in Tables 2 and 3 respectively and the Fischer-Clifford matrices of $\bar{G} = 2^{11}{:}M_{24}$ given in Table 7, the full character table of $\bar{G}$ can then be constructed. The character table of $\bar{G} = 2^{11}{:}M_{24}$ is partitioned row-wise into 3 blocks, where each block corresponds to an inertia factor group H_i. Given the character table of the inertia factor group H_i, we take the columns of this character table and multiply them by the corresponding rows of the Fischer-Clifford matrix denoted by $M(g)$ and then we fill the portions of the character table of $\bar{G}$ for its classes which come from the coset Ng. That is, we multiply the appropriate partial character tables of the inertia factor groups by the corresponding rows of the Fischer matrices to obtain the full character table of $\bar{G}$. Again, by Theorem 3.1, since $\bar{G}$ has 80 conjugacy classes, it must have 80 irreducible characters. For instance, on the identity of G we have

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 276 & 20 & -12 \\ 1771 & -21 & 11 \end{pmatrix}.$$

Multiplying each row of M(1A) by 1st columns of the corresponding Tables 4, 5 and 6 (Character tables of inertia factor groups; H_1, H_2 and H_3), we obtain the values of the characters of $\bar{G}$ on $\bar{G}$-classes $1A$, $2A$ and $2B$ and able to fill the 1st portions of the 1st, 2nd and third blocks of the character table of $\bar{G}$ as follows:

1. If column 1 of Table 1.3 is multiplied by row 1 of M(1A), we get:

$$\begin{pmatrix} 1 \\ 23 \\ 45 \\ 45 \\ 231 \\ 231 \\ 252 \\ 253 \\ 483 \\ 770 \\ 770 \\ 990 \\ 990 \\ 1035 \\ 1035 \\ 1035 \\ 1265 \\ 1771 \\ 2024 \\ 2277 \\ 3312 \\ 3520 \\ 5313 \\ 5544 \\ 5796 \\ 10395 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 23 & 23 & 23 \\ 45 & 45 & 45 \\ 45 & 45 & 45 \\ 231 & 231 & 231 \\ 231 & 231 & 231 \\ 252 & 252 & 252 \\ 253 & 253 & 253 \\ 483 & 483 & 483 \\ 770 & 770 & 770 \\ 770 & 770 & 770 \\ 990 & 990 & 990 \\ 990 & 990 & 990 \\ 1035 & 1035 & 1035 \\ 1035 & 1035 & 1035 \\ 1035 & 1035 & 1035 \\ 1265 & 1265 & 1265 \\ 1771 & 1771 & 1771 \\ 2024 & 2024 & 2024 \\ 2277 & 2277 & 2277 \\ 3312 & 3312 & 3312 \\ 3520 & 3520 & 3520 \\ 5313 & 5313 & 5313 \\ 5544 & 5544 & 5544 \\ 5796 & 5796 & 5796 \\ 10395 & 10395 & 10395 \end{pmatrix}$$

2. If column 1 of Table 1.4 is multiplied by row 2 of M(1A), we get

$$\begin{pmatrix} 1\\ 1\\ 21\\ 21\\ 45\\ 45\\ 45\\ 45\\ 55\\ 99\\ 99\\ 154\\ 154\\ 210\\ 210\\ 231\\ 231\\ 231\\ 385\\ 385\\ 560 \end{pmatrix} \times \begin{pmatrix} 276 & 20 & -12 \end{pmatrix} = \begin{pmatrix} 276 & 20 & -12\\ 276 & 20 & -12\\ 5796 & 420 & -252\\ 5796 & 420 & -252\\ 12420 & 900 & -540\\ 12420 & 900 & -540\\ 12420 & 9002 & -540\\ 12420 & 900 & -540\\ 15180 & 1100 & -660\\ 15180 & 1100 & -660\\ 27324 & 1980 & -1188\\ 27324 & 1980 & -1188\\ 42504 & 3080 & -1848\\ 42504 & 3080 & -1848\\ 57960 & 4200 & -2520\\ 57960 & 4200 & -2520\\ 63756 & 4620 & -2772\\ 63756 & 4620 & -2772\\ 154560 & 11200 & -6720\\ 106260 & 7700 & -4620\\ 106260 & 7700 & -4620 \end{pmatrix}$$

3. If column 1 of Table 1.5 is multiplied by row 3 of M(1A), we have that

$$\begin{pmatrix} 1\\ 1\\ 5\\ 5\\ 5\\ 5\\ 6\\ 6\\ 9\\ 9\\ 10\\ 10\\ 12\\ 16\\ 18\\ 18\\ 18\\ 30\\ 45\\ 45\\ 45\\ 45\\ 72\\ 72\\ 90\\ 90\\ 90\\ 90\\ 108\\ 135\\ 135\\ 135\\ 135 \end{pmatrix} \times \begin{pmatrix} 1771 & -21 & 11 \end{pmatrix} = \begin{pmatrix} 1771 & -21 & 11\\ 1771 & -21 & 11\\ 8855 & -105 & 55\\ 8855 & -105 & 55\\ 8855 & -105 & 55\\ 8855 & -105 & 55\\ 15939 & -189 & 99\\ 15939 & -189 & 99\\ 17710 & -210 & 110\\ 17710 & -210 & 110\\ 28336 & -336 & 176\\ 10626 & -126 & 66\\ 10626 & -126 & 66\\ 21252 & -252 & 132\\ 31878 & -378 & 198\\ 53130 & -630 & 330\\ 31878 & -378 & 198\\ 31878 & -378 & 198\\ 79695 & -945 & 495\\ 79695 & -945 & 495\\ 79695 & -945 & 495\\ 79695 & -945 & 495\\ 127512 & -1512 & 792\\ 127512 & -1512 & 792\\ 159390 & -1890 & 990\\ 159390 & -1890 & 990\\ 159390 & -1890 & 990\\ 159390 & -1890 & 990\\ 191268 & -2268 & 118\\ 239085 & -2835 & 1485\\ 239085 & -2835 & 1485\\ 239085 & -2835 & 1485\\ 239085 & -2835 & 1485 \end{pmatrix}$$

Carrying out this operation on all other classes of G, we are able to obtain all the 80 irreducible characters of $\bar{G}$ given in Table 8. The Table is divided into three blocks corresponding to three inertia groups $\bar{G}=\bar{H}_1$, $\bar{H}_2$ and $\bar{H}_3$. The consistency of the Character Table was checked using **Programme E**.

6. Fusion of $\bar{G} = 2^{11}{:}M_{24}$ into Co_1

We first state and prove the following theorem.

Theorem 6.1 ([15]). *Let ϕ be a character of H and let $\phi\uparrow_H^G$ be the induced character from H to G. Let $g \in G$ and suppose that $[g]$ breaks into m classes in H with representatives $x_1, x_2, \dots, x_m$. If $H \cap [g] = \emptyset$, then $\phi\uparrow_H^G(g) = 0$, while if $H \cap [g] \neq \emptyset$, then*

$$\phi\uparrow_H^G(g) = |C_G(g)| \sum_{i=1}^{m} \frac{\phi(x_i)}{|C_H(x_i)|}.$$

Proof. We have

$$\phi\uparrow_H^G(g) = \frac{1}{|H|} \sum_{x \in G} \phi^0(xgx^{-1}).$$

If $H \cap [g] = \emptyset$, then $xgx^{-1} \notin H, \forall x \in G$ and thus $\phi^0(xgx^{-1}) = 0$ and we have that $\phi\uparrow_H^G(g) = 0$.
Now if $H \cap [g] \neq \emptyset$, then let $h \in H \cap [g]$. As x runs over G, we have $xgx^{-1} = h$ for exactly $|C_G(g)|$ times, so $\phi\uparrow_H^G(g) = \frac{|C_G(g)|}{|H|} \sum_{y \in [g]} \phi^0(y)$. Now $\phi^0(y) = 0$ if $y \notin H$ and $[g] \cap H$ contains $[H : C_H(x_i)]$ conjugates of each x_i. Therefore

$$\phi\uparrow_H^G(g) = |C_G(g)| \sum_{i=1}^{m} \frac{\phi(x_i)}{|C_H(x_i)|}.$$

□

Corollary 6.2. *Let $H \leq G$. Let $g \in G$ and $x_1, x_2, \dots, x_m$ be representatives of the conjugacy classes of H that fuse to $[g]$. Then*

$$\chi(G/H)(g) = \sum_{i=1}^{m} \frac{|C_G(g)|}{|C_H(x_i)|}.$$

Proof. This follows from Theorem 5.1 □

We now use the results of Theorem 6.1 and Corollary 6.2 to fuse the classes of $\bar{G}$ into the classes of Co_1. Let $\chi(Co_1|\bar{G})$ denote the permutation character of CO_1 on the cosets of $\bar{G}$ in Co_1. Let $[g]_{Co_1}$ denote a conjugacy class of Co_1 with representative g and similarly $[\bar{g}]_{\bar{G}}$ denote a conjugacy class of $\bar{G}$ with representative $\bar{g}$. We recall that

$$\begin{cases} \text{if } \bar{G} \cap [g]_{Co_1} = \emptyset, \text{ then } \chi(Co_1|\bar{G})(g) = 0 \\ \text{if } \bar{G} \cap [g]_{Co_1} \neq \emptyset, \text{ then } \chi(Co_1|\bar{G})(g) = \sum_{i=1}^{r} \frac{|C_{CO_1}(g)|}{|C_{\bar{G}}(x_i)|}, \end{cases} \tag{1}$$

where $\{x_1, x_2, \cdots, x_r\}$ is the set of class representatives of $\bar{G}$ that fuse to g in Co_1. Thus knowledge of the permutation character values on the classes of Co_1 will assist to know the possibilities for the fusion of classes of $\bar{G}$ to classes of Co_1. On the class where the permutation character value is zero, then no class of $\bar{G}$ fuses to it.
Let $[g_1]$ and $[g_2]$ be two arbitrary conjugacy classes of Co_1 such that $g_1^p \in [g_2]$ for some prime p. Let $[\bar{g}_1]$ and $[\bar{g}_2]$ be two arbitrary conjugacy classes of $\bar{G}$ such that $\bar{g}_1^p \in [\bar{g}_2]$ for some prime p. If the class $[\bar{g}_1]$ fuses to $[g_1]$ then clearly the class $[\bar{g}_2]$ must fuse to $[g_2]$. Thus knowledge of the power maps is also important in producing partial fusions and removing some ambiguities. The power maps of elements of Co_1 and its permutation character of degree 8292375 are given in the Atlas [11]. We give the permutation character below:

$$\chi(Co_1|\bar{G}) = 1\chi_1 + 17250\chi_6 + 80730\chi_{10} + 644644\chi_{16} + 2055625\chi_{25} + 5494125\chi_{32}.$$

Using the information provided by the conjugacy classes of elements of $2^{11}{:}M_{24}$ and Co_1 in the Atlas, the power maps and the permutation character of Co_1 of degree 8292375, we are able to obtain all the fusions of $\bar{G}$ into Co_1.

For example, obtaining the Character Table of Co_1 in GAP, we see that the character value of the identity class $1A$ of Co_1 is:

$$\chi_1 + \chi_6 + \chi_{10} + \chi_{16} + \chi_{25} + \chi_{32} = 1 + 17250 + 80730 + 644644 + 2055625 + 5494125 = 8292375.$$

Again with the help of GAP, we see that $|C_{Co_1}(1A)| = 4157776806543360000$ and $|C_{\bar{G}}(1A)| = 501397585920$. Now,

$$\frac{|C_{Co_1}(1A)|}{|C_{\bar{G}}(1A)|} = \frac{4157776806543360000}{501397585920} = 8292375.$$

This shows that $1A$ of $\bar{G}$ fuses into $1A$ of Co_1. Similarly, the character value of $2A$ of Co_1 is

$\chi_1 + \chi_6 + \chi_{10} + \chi_{16} + \chi_{25} + \chi_{32} = 1 + 610 + 1626 + 3108 + 7625 + 19565 = 32535$ and $|C_{Co_1}(2A)| = 89181388800$.

We also have that $|C_{\bar{G}}(2A)| = 660602880$ and $|C_{\bar{G}}(2C)| = 2752512$. Thus,

$$\frac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2A)|} = \frac{89181388800}{660602880} = 135 \text{ and } \frac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2C)|} = \frac{89181388800}{2752512} = 32400.$$

We see that $\frac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2A)|} + \frac{|C_{Co_1}(2A)|}{|C_{\bar{G}}(2C)|} = 135 + 32400 = 32535$, which is the character value of $2A$ of Co_1 we found. This means that both $2A$ and $2C$ of $\bar{G}$ fuse into $2A$ of Co_1. Carrying out this operation on all the conjugacy classes of $\bar{G}$, we are able to identify which classes of Co_1 they fuse to. However, in some cases, having only the information about the conjugacy classes, power maps, permutation character values, may not be enough to complete the fusion map of certain classes due to some possibilities that arise. To obtain the full fusions in such cases, we restrict irreducible characters of the mother group of small degree to $\bar{G}$. In determining the restrictions of irreducible characters of the mother group to $\bar{G}$, we apply the technique of set intersections for characters. The concept of set intersections for characters has not been demonstrated in this paper but for more information about this concept (intersection of characters), readers are refered to [7], [6], [9] and [13].

7. GAP Programmes and Tables

The following are GAP programmes and Tables that have been cited in the main work.

Programme A

```
work :=[];;
output:=[];;
work[1]:=input[1];;
work[2]:=input[2];;
work[3]:=work[1] * work[2];;
work[4]:=work[3] * work[2];;
work[5]:=work[3] * work[4];;
work[6]:=work[3] * work[5];;
work[7]:=work[6] * work[3];;
work[8]:= work[6] * work[7];;
work[7]:=work[8] * work[8];;
work[2]:= work[7] * work[7];;
work[6]:=work[5] * work[5];;
work[1] := work[5] * work[6];;
work[5]:= work[4] * work[4];;
work[6] := work[4] * work[5];;
work[7]:= work[5] * work[6];;
work[8] := work[7]^-1;;
work[6] := work[8] * work[2];;
work[2] := work[6] * work[7];;
work[4] := work[3] * work[3];;
work[5] := work[3] * work[4];;
work[6] := work[5] * work[5];;
```

```
work[7] := work[6]^-1;;
work[8] := work[7] * work[1];;
work[1] := work[8] * work[6];;
output[1] := work[1];;
output[2] := work[2];;
```

Programme D

```
C:=List(ConjugacyClasses(G),Representative);;
M:=[]; g:=C[i];
for n in N do
Add(M, n*g*Inverse(n)*Inverse(g)); od;
M:=AsGroup(M);
cent:=Centralizer(G, g);;
O:=RightCosets(N,M);; D:=O;; B:=[];
for j in [1..Size(N)] do B[j]:=[];od; j:=1; while D <> [] do
x:=Representative(D[1]); for i in [1..Size(O)] do y:=x^cent; if
Intersection(y, O[i]) <> [] then Add(B[j],O[i]); fi;od;
D:=Difference(D,B[j]); j:=j+1; od; i:=1; while B[i] <> [] do
Print(Size(B[i]));Print(" - " );i:=i+1;
od;
I:=Irr(N);; IM:=[]; for i in [1..Size(I)] do if
IsSubgroup(Kernel(I[i]), M) then Add(IM,I[i]); fi; od;
oo:=Orbits(cent,IM);;
FM:=[];;
for i in [1..Size(oo)] do
Append(FM,[AsList(Sum(oo[i]))]);
od;
M1:=TransposedMat(FM);; M2:=AsDuplicateFreeList(M1);;
FM:=TransposedMat(M2);; Display(FM);
```

Programme E

```
K:=function()local K;;
K:=rec();;
K.SizesCentralizers:=[];;
K.OrdersClassRepresentatives:=[];;
K.Irr:=[[ ],[ ],[ ],...[ ];;
K.UnderlyingCharacteristic:=0;;
ConvertToCharacterTable(K);;
return K;;
end;K:=K();;
SetInfoLevel(InfoCharacterTable,2);;
IsInternallyConsistent(K);
true
PossiblePowerMaps(K,p);
```

Programme F

```
for j in [1..4] do
for k in [2..4] do
for l in [3..4] do
for m in [4..4] do
if c'*gens[1]*Inverse(c')=gens[j] then
Print(j, " ");
elif c'*gens[1]*Inverse(c')=gens[j]*gens[k] then
Print(j, " ");
Print(k, " ");
```

```
elif c'*gens[1]*Inverse(c')=gens[j]*gens[k]*gens[l] then
Print(j, " ");
Print(k, " ");
Print(l, " ");
elif c'*gens[1]*Inverse(c')=gens[j]*gens[k]*gens[l]*gens[m] then
Print (j, " ");
Print (k, " ");
Print (l, " ");
Print (m, " ");
fi;
od;od;od;od;od;
```

Programme G

```
I:=Irr(N);;
o:=Orbits(G,I);;
M:=[AsList(Sum(o[1])),AsList(Sum(o[2]))\cdots,AsList(Sum(o[Size(o)]))];;
M1:=TransposedMat(M);;
M2:=AsDuplicateFreeList(M1);;
FM:=TransposedMat(M2);;
```

Table 1: Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$

$[g]_G$	k	f_j	m_i	$[g]_{\bar{G}}$	$\lvert C_{\bar{G}}(g)\rvert$	2	3	5	7	Size of $[g]_{\bar{G}}$
$1A$	2048	$f_1 = 1$	$m_1 = 1$	$1A$	501397585920	$1A$	$1A$	$1A$	$1A$	1
		$f_2 = 759$	$m_2 = 759$	$2A$	660602880	$1A$	$2A$	$2A$	$2A$	759
		$f_3 = 1288$	$m_3 = 1288$	$2B$	389283840	$1A$	$2B$	$2B$	$2B$	1288
$2A$	64	$f_1 = 1$	$m_1 = 32$	$2E$	491520	$1A$	$2E$	$2E$	$2E$	1020096
		$f_2 = 32$	$m_2 = 1024$	$2F$	15360	$1A$	$2F$	$2F$	$2F$	32643072
		$f_3 = 15$	$m_3 = 480$	$4E$	32768	$2A$	$4E$	$4E$	$4E$	15301440
		$f_4 = 15$	$m_4 = 480$	$4F$	32768	$2A$	$4F$	$4F$	$4F$	15301440
		$f_5 = 1$	$m_5 = 32$	$4G$	491520	$2B$	$4G$	$4G$	$4G$	1020096
$2B$	128	$f_1 = 1$	$m_1 = 16$	$2C$	344064	$1A$	$2C$	$2C$	$2C$	1457280
		$f_2 = 56$	$m_2 = 896$	$2D$	6144	$1A$	$2D$	$2D$	$2D$	81607680
		$f_3 = 56$	$m_3 = 896$	$4A$	6144	$2A$	$4A$	$4A$	$4A$	81607680
		$f_4 = 1$	$m_4 = 16$	$4B$	344064	$2A$	$4B$	$4B$	$4B$	1457280
		$f_5 = 7$	$m_5 = 112$	$4C$	49152	$2A$	$4C$	$4C$	$4C$	10200960
		$f_6 = 7$	$m_6 = 112$	$4D$	49152	$2A$	$4D$	$4D$	$4D$	10200960
$3A$	32	$f_1 = 1$	$m_1 = 64$	$3A$	34560	$3A$	$1A$	$3A$	$3A$	14508032
		$f_2 = 15$	$m_2 = 960$	$6A$	2304	$3A$	$2A$	$6A$	$6A$	217620480
		$f_3 = 10$	$m_3 = 640$	$6B$	3456	$3A$	$2A$	$6B$	$6B$	145080320
		$f_4 = 6$	$m_4 = 384$	$6C$	5760	$3A$	$2B$	$6C$	$6C$	87048192
$3B$	8	$f_1 = 1$	$m_1 = 256$	$3B$	4032	$3B$	$1A$	$3B$	$3B$	124354560
		$f_2 = 7$	$m_2 = 896$	$6D$	576	$3B$	$2B$	$6D$	$6D$	870481920
$4A$	16	$f_1 = 1$	$m_1 = 128$	$4K$	2048	$2C$	$8C$	$4K$	$4K$	244823040
		$f_2 = 2$	$m_2 = 256$	$4L$	1024	$2C$	$4K$	$4L$	$4L$	489646080
		$f_3 = 8$	$m_3 = 1024$	$8D$	256	$4B$	$8D$	$8D$	$8D$	1958584320

Table 1: Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$ (continued)

$[g]_G$	k	f_j	m_i	$[g]_{\bar{G}}$	$\lvert C_{\bar{G}}(g)\rvert$	2	3	5	7	Size of $[g]_{\bar{G}}$
		$f_4 = 1$	$m_4 = 128$	$8E$	2048	$4B$	$8E$	$8E$	$8E$	244823040
		$f_5 = 2$	$m_5 = 256$	$4M$	1024	$2D$	$4M$	$4M$	$4M$	489646080
		$f_6 = 1$	$m_5 = 128$	$8F$	2048	$4A$	$8F$	$8F$	$8F$	244823040
		$f_7 = 1$	$m_5 = 128$	$8G$	2048	$4D$	$8G$	$8G$	$8G$	244823040
$4B$	8	$f_1 = 1$	$m_1 = 256$	$4N$	768	$2E$	$4N$	$4N$	$4N$	652861440
		$f_2 = 3$	$m_2 = 768$	$8H$	256	$4E$	$8H$	$8H$	$8H$	1958584320
		$f_3 = 3$	$m_3 = 768$	$4O$	256	$2F$	$4O$	$4O$	$4O$	1958584320
		$f_4 = 1$	$m_4 = 256$	$8I$	768	$4F$	$8I$	$8I$	$8I$	652861440
$4C$	16	$f_1 = 1$	$m_1 = 128$	$4H$	6144	$2C$	$4H$	$4H$	$4H$	81607680
		$f_2 = 6$	$m_2 = 768$	$4I$	1024	$2C$	$4I$	$4I$	$4I$	489646080
		$f_3 = 1$	$m_3 = 128$	$8A$	6144	$4A$	$8A$	$8A$	$8A$	81607680
		$f_4 = 6$	$m_2 = 768$	$8B$	1024	$4A$	$8B$	$8B$	$8B$	489646080
		$f_5 = 1$	$m_2 = 128$	$4J$	6144	$2D$	$4J$	$4J$	$4J$	81607680
		$f_6 = 1$	$m_2 = 128$	$8C$	6144	$4B$	$8C$	$8C$	$8C$	81607680
$5A$	8	$f_1 = 1$	$m_1 = 256$	$5A$	2480	$5A$	$5A$	$1A$	$5A$	2021764459
		$f_2 = 3$	$m_2 = 768$	$10A$	160	$5A$	$10A$	$2A$	$10A$	3133734912
		$f_3 = 4$	$m_3 = 1024$	$10B$	120	$5A$	$10B$	$2B$	$10B$	4178313216
$6A$	4	$f_1 = 1$	$m_1 = 512$	$6G$	96	$3B$	$2E$	$6G$	$6G$	5222891520
		$f_2 = 2$	$m_2 = 1024$	$6H$	48	$3B$	$2F$	$6H$	$6H$	10445783040
		$f_3 = 1$	$m_3 = 512$	$12E$	96	$6D$	$4G$	$12E$	$12E$	10445783040
$6B$	8	$f_1 = 1$	$m_1 = 256$	$6E$	192	$3A$	$2C$	$6E$	$6E$	5222891520
		$f_2 = 2$	$m_2 = 512$	$6F$	96	$3A$	$2D$	$6F$	$6F$	2611445760
		$f_3 = 1$	$m_3 = 256$	$12A$	192	$6A$	$4A$	$12A$	$12A$	5222891520
		$f_4 = 2$	$m_2 = 512$	$12B$	96	$6A$	$4B$	$12B$	$12B$	2611445760
		$f_5 = 1$	$m_3 = 256$	$12C$	192	$6A$	$4C$	$12C$	$12C$	5222891520
		$f_6 = 1$	$m_3 = 256$	$12D$	192	$6A$	$4D$	$12D$	$12D$	5222891520
$7A$	4	$f_1 = 1$	$m_1 = 512$	$7A$	168	$7A$	$7B$	$7B$	$1A$	2984509440
		$f_2 = 3$	$m_2 = 1536$	$14A$	56	$7A$	$14B$	$14B$	$2A$	8953528320
$7B$	4	$f_1 = 1$	$m_1 = 512$	$7B$	168	$7B$	$7A$	$7A$	$1A$	2984509440
		$f_2 = 3$	$m_2 = 1536$	$14B$	56	$7B$	$14A$	$14A$	$2A$	8953528320
$8A$	4	$f_1 = 1$	$m_1 = 128$	$8J$	256	$4K$	$8J$	$8J$	$8J$	1958584320
		$f_2 = 1$	$m_2 = 128$	$8K$	128	$4L$	$8K$	$8K$	$8K$	1958584320
		$f_3 = 1$	$m_3 = 128$	$16A$	32	$8E$	$16A$	$16A$	$16A$	1958584320
		$f_4 = 1$	$m_4 = 128$	$16B$	256	$1A$	$16B$	$16B$	$16B$	1958584320
$10A$	4	$f_1 = 1$	$m_1 = 512$	$10C$	80	$5A$	$10C$	$2E$	$10C$	6267469824
		$f_2 = 2$	$m_2 = 1024$	$10D$	40	$5A$	$10D$	$2F$	$10D$	12534939650
		$f_3 = 1$	$m_3 = 512$	$20A$	80	$10B$	$20A$	$4G$	$20A$	6267469824
$11A$	2	$f_1 = 1$	$m_1 = 1024$	$11A$	22	$11A$	$11A$	$11A$	$11A$	22790799360
		$f_2 = 1$	$m_2 = 1024$	$22A$	22	$11A$	$22A$	$22A$	$22A$	22790799360

Table 1: Conjugacy Classes of $\bar{G} = 2^{11}{:}M_{24}$ (continued)

$[g]_G$	k	f_j	m_i	$[g]_{\bar{G}}$	$\lvert C_{\bar{G}}(g)\rvert$	2	3	5	7	Size of $[g]_{\bar{G}}$
$12A$	4	$f_1 = 1$	$m_1 = 512$	$12F$	48	$6E$	$4H$	$12F$	$12A$	10445783040
		$f_2 = 1$	$m_2 = 512$	$24A$	48	$12A$	$8B$	$24A$	$24A$	10445783040
		$f_3 = 1$	$m_3 = 512$	$12G$	48	$6E$	$4I$	$12G$	$12G$	10445783040
		$f_4 = 1$	$m_4 = 512$	$24B$	48	$12A$	$8A$	$24B$	$24B$	10445783040
$12B$	2	$f_1 = 1$	$m_1 = 1024$	$12H$	24	$6G$	$4N$	$12H$	$12H$	20981566080
		$f_2 = 1$	$m_2 = 1024$	$12I$	24	$6H$	$4O$	$12I$	$12I$	20981566080
$14A$	2	$f_1 = 1$	$m_1 = 1024$	$14C$	28	$7A$	$14D$	$14D$	$2C$	17907056640
		$f_2 = 1$	$m_2 = 1024$	$28A$	28	$14A$	$28B$	$28B$	$4A$	17907056640
$14B$	2	$f_1 = 1$	$m_1 = 1024$	$14D$	28	$7B$	$14C$	$14C$	$2C$	17907056640
		$f_2 = 1$	$m_2 = 1024$	$28B$	28	$14B$	$28A$	$28A$	$4A$	17907056640
$15A$	2	$f_1 = 1$	$m_1 = 1024$	$15A$	30	$15A$	$5A$	$3A$	$15B$	16713252860
		$f_2 = 1$	$m_2 = 1024$	$30A$	30	$15A$	$10A$	$6B$	$30B$	16713252860
$15B$	2	$f_1 = 1$	$m_1 = 1024$	$15B$	30	$15B$	$5A$	$3A$	$15A$	16713252860
		$f_2 = 1$	$m_2 = 1024$	$30B$	30	$15B$	$10A$	$6B$	$30A$	16713252860
$21A$	1	$f_1 = 1$	$m_1 = 2048$	$21A$	21	$21A$	$7B$	$21B$	$3B$	23876075520
$21B$	1	$f_1 = 1$	$m_1 = 2048$	$21B$	21	$21B$	$7A$	$21A$	$3B$	23876075520
$23A$	1	$f_1 = 1$	$m_1 = 2048$	$23A$	23	$23A$	$23A$	$23B$	$23B$	21799895040
$23B$	1	$f_1 = 1$	$m_1 = 2048$	$23B$	23	$23B$	$23B$	$23A$	$23A$	21799895040

Table 2: Fusion of the Classes from H_2 to $G = M_{24}$

$[h]_{H_2} \mapsto$	$[g]_G$	$[h]_{H_2} \mapsto$	$[g]_G$	$[h]_{H_2} \mapsto$	$[g]_G$	$[h]_{H_2} \mapsto$	$[g]_G$
$1a$	$1A$	$4a$	$4A$	$6a$	$6B$	$8b$	$8A$
$2c$	$2A$	$4b$	$4A$	$6b$	$6B$	$10a$	$10a$
$2a$	$2B$	$4d$	$4A$	$7a$	$7A$	$11a$	$11A$
$2b$	$2B$	$4c$	$4C$	$7b$	$7B$	$12a$	$12A$
$3a$	$3A$	$5a$	$5A$	$8a$	$8A$	$14a$	$14A$

Table 3: Fusion of the Classes from H_3 to $G = M_{24}$

$[h]_{H_3} \mapsto$	$[g]_G$	$[h]_{H_3} \mapsto$	$[g]_G$	$[h]_{H_3} \mapsto$	$[g]_G$	$[h]_{H_3} \mapsto$	$[g]_G$	$[h]_{H_3} \mapsto$	$[g]_G$	$[h]_{H_3} \mapsto$	$[g]_G$
$1a$	$1A$	$2c$	$2B$	$4a$	$4A$	$4i$	$4B$	$5A$	$5A$	$6c$	$6B$
$2b$	$2A$	$2e$	$2B$	$4c$	$4A$	$4b$	$4C$	$6d$	$6A$	$8a$	$8A$
$2d$	$2A$	$3a$	$3A$	$4f$	$4A$	$4d$	$4C$	$6e$	$6A$	$10a$	$10A$
$2f$	$2A$	$3b$	$3A$	$4e$	$4B$	$4g$	$4C$	$6a$	$6B$	$12a$	$12A$
$2a$	$2B$	$3c$	$3B$	$4h$	$4B$	$4j$	$4C$	$6b$	$6B$	$12b$	$12A$
$12c$	$12B$	$15b$	$15A$	$15a$	$15B$						

Table 4: Character Table $H_1 = G = M_{24}$

$[g]$	$1A$	$3A$	$5A$	$15A$	$15B$	$2A$	$10A$	$3B$	$6A$	$2B$	$4A$	$8A$	$4B$	$7A$	$7B$	$21A$	$21B$	$14A$	$14B$	$4C$	$6B$	$12A$	$11A$	$23A$	$23B$	$12B$
$\lvert C_{H_1}(g)\rvert$	244823040	1080	60	15	15	7680	20	504	24	21504	128	16	96	42	42	21	21	14	14	384	24	12	11	23	23	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	23	5	3	0	0	-1	-1	-1	-1	7	3	1	-1	2	2	-1	-1	0	0	-1	1	-1	1	0	0	-1
χ_3	45	0	0	0	0	5	0	3	-1	-3	1	-1	1	B	$/B$	$/B$	B	$-/B$	$-B$	-3	0	0	1	-1	-1	1
χ_4	45	0	0	0	0	5	0	3	-1	-3	1	-1	1	$/B$	B	B	$/B$	$-B$	$-/B$	-3	0	0	1	-1	-1	1
χ_5	231	-3	1	A	$/A$	-9	1	0	0	7	-1	-1	3	0	0	0	0	0	0	-1	1	-1	0	1	1	0
χ_6	231	-3	1	$/A$	A	-9	1	0	0	7	-1	-1	3	0	0	0	0	0	0	-1	1	-1	0	1	1	0
χ_7	252	9	2	-1	-1	12	2	0	0	28	4	0	0	0	0	0	0	0	0	4	1	1	-1	-1	-1	0
χ_8	253	10	3	0	0	-11	-1	1	1	13	1	-1	1	1	1	1	1	-1	-1	-3	-2	0	0	0	0	1
χ_9	483	6	-2	1	1	3	-2	0	0	35	3	-1	3	0	0	0	0	0	0	3	2	0	-1	0	0	0
χ_{10}	770	5	0	0	0	10	0	-7	1	-14	-2	0	-2	0	0	0	0	0	0	2	1	-1	0	D	$/D$	1
χ_{11}	770	5	0	0	0	10	0	-7	1	-14	-2	0	-2	0	0	0	0	0	0	2	1	-1	0	$/D$	D	1
χ_{12}	990	0	0	0	0	-10	0	3	-1	-18	2	0	-2	B	$/B$	$/B$	B	$/B$	B	6	0	0	0	1	1	1
χ_{13}	990	0	0	0	0	-10	0	3	-1	-18	2	0	-2	$/B$	B	B	$/B$	B	$/B$	6	0	0	0	1	1	1
χ_{14}	1035	0	0	0	0	35	0	6	2	27	-1	1	3	-1	-1	-1	-1	-1	-1	3	0	0	1	0	0	0
χ_{15}	1035	0	0	0	0	-5	0	-3	1	-21	3	-1	-1	C	$/C$	$-/B$	$-B$	0	0	3	0	0	1	0	0	-1
χ_{16}	1035	0	0	0	0	-5	0	-3	1	-21	3	-1	-1	$/C$	C	$-B$	$-/B$	0	0	3	0	0	1	0	0	-1
χ_{17}	1265	5	0	0	0	-15	0	8	0	49	1	1	-3	-2	-2	1	1	0	0	1	1	0	0	0	0	-1
χ_{18}	1771	16	1	1	1	11	1	7	-1	-21	-5	-1	-1	0	0	0	0	0	0	-7	1	-1	0	0	0	0
χ_{19}	2024	-1	-1	-1	-1	24	-1	8	0	8	0	0	0	1	1	1	1	1	0	3	0	0	0	0	0	-1
χ_{20}	2277	0	-3	0	0	-19	1	6	2	21	1	-1	-3	2	2	-1	-1	0	0	-3	0	0	0	0	0	0
χ_{21}	3312	0	-3	0	0	16	1	-6	-2	48	0	0	0	1	1	1	1	-1	-1	0	0	0	1	0	0	0
χ_{22}	3520	10	0	0	0	0	0	-8	0	64	0	0	0	-1	-1	-1	-1	1	1	0	-2	0	0	1	1	0
χ_{23}	5313	-15	3	0	0	9	-1	0	0	49	-3	-1	-3	0	0	0	0	0	0	1	1	1	0	0	0	0
χ_{24}	5544	9	-1	-1	-1	24	-1	0	0	-56	0	0	0	0	0	0	0	0	0	-8	1	1	0	1	1	0
χ_{25}	5796	-9	1	1	1	36	1	0	0	-28	4	0	0	0	0	0	0	0	0	-4	-1	-1	-1	0	0	0
χ_{26}	10395	0	0	0	0	-45	0	0	0	-21	-1	1	3	0	0	0	0	0	0	3	0	0	0	-1	-1	0

Table 5: Character Table of $H_2 = M_{22}{:}2$

$[g]$	$1a$	$2a$	$3a$	$4a$	$6a$	$12a$	$4b$	$2b$	$4c$	$8a$	$4d$	$8b$	$7a$	$7b$	$2c$	$14a$	$14b$	$5a$	$10a$	$11a$	$6b$
$\|C_{H_2}(g)\|$	887040	768	72	64	24	12	64	7680	96	16	32	16	14	14	640	14	14	10	10	11	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	1	-1	1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	-1	1	-1
χ_3	21	5	3	-1	-1	-1	1	-1	1	1	3	-1	0	0	7	0	0	1	-1	-1	1
χ_4	21	5	3	1	-1	1	1	1	1	-1	-3	-1	0	0	-7	0	0	1	1	-1	-1
χ_5	45	-3	0	-3	0	0	1	5	1	-1	1	-1	A	$/A$	-3	$-A$	$-/A$	0	0	1	0
χ_6	45	-3	0	-3	0	0	1	5	1	-1	1	-1	$/A$	A	-3	$-/A$	$-A$	0	0	1	0
χ_7	45	-3	0	3	0	0	1	-5	1	1	-1	-1	A	$/A$	3	A	$/A$	0	0	1	0
χ_8	45	-3	0	3	0	0	1	-5	1	1	-1	-1	$/A$	A	3	$/A$	A	0	0	1	0
χ_9	55	7	1	1	1	1	-1	5	3	-1	1	1	-1	-1	13	-1	-1	0	0	0	1
χ_{10}	99	3	0	3	0	0	-1	-1	3	-1	-1	-1	1	1	15	1	1	0	0	0	-1
χ_{11}	99	3	0	-3	0	0	-1	1	3	1	1	-1	1	1	-15	-1	-1	-1	-1	0	0
χ_{12}	154	10	1	2	1	-1	2	6	-2	0	2	0	0	0	14	0	0	-1	1	0	0
χ_{13}	154	10	1	-2	1	1	2	-6	-2	0	-2	0	0	0	-14	0	0	-1	1	0	-1
χ_{14}	210	2	3	-2	-1	1	-2	-10	-2	0	2	0	0	0	14	0	0	-1	-1	0	1
χ_{15}	210	2	3	2	-1	-1	-2	10	-2	0	-2	0	0	0	-14	0	0	0	0	1	-1
χ_{16}	231	7	-3	-1	1	-1	-1	-9	-1	-1	-1	-1	0	0	7	0	0	0	0	1	1
χ_{17}	231	7	-3	-1	1	-1	-1	-9	-1	-1	-1	-1	0	0	7	0	0	1	1	0	1
χ_{18}	231	7	-3	1	1	1	-1	9	-1	1	1	-1	0	0	-7	0	0	1	-1	0	-1
χ_{19}	385	1	-2	-3	-2	0	1	5	1	1	-3	1	0	0	21	0	0	0	0	0	0
χ_{20}	385	1	-2	3	-2	0	1	-5	1	-1	3	1	0	0	-21	0	0	0	0	0	0
χ_{21}	560	-16	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0

In Tables 4, 5 and 6, we take $A = \frac{-1-\sqrt{-15}}{2}$, $B = \frac{-1-\sqrt{-7}}{2}$, $C = -1-\sqrt{-7}$ and $D = \frac{-1-\sqrt{-23}}{2}$, where $/A$, $/B$, $/C$ and $/D$ are conjugates of A, B, C and D respectively.

Table 6: Character Table of $H_3 = 2^6{:}(3.A_6){:}2$

$[g]$	$1a$	$2a$	$2b$	$3a$	$3b$	$6a$	$4a$	$2c$	$4b$	$2d$	$6b$	$10a$	$5a$	$15a$	$15b$	$4c$	$2e$	$4d$	$8a$	$4e$	$4f$	$12a$	$6c$	$12b$	$4g$	$4h$	$6d$	$3c$	$4i$	$4j$	$2f$	$6e$	$12c$
$\|C_{H_3}(g)\|$	138240	3072	7680	1080	72	24	128	768	128	256	24	20	60	15	15	64	384	384	16	32	32	12	12	12	96	32	24	72	96	128	384	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
χ_3	5	5	5	5	−1	−1	1	1	1	1	1	0	0	0	0	1	1	1	−1	−1	−1	1	1	−1	−1	−1	2	2	−3	−3	−3	0	0
χ_4	5	5	5	5	2	2	1	1	1	1	1	0	0	0	0	−3	−3	−3	−1	−1	−1	0	0	−1	−1	−1	−1	−1	1	1	1	1	1
χ_5	5	5	5	5	−1	−1	1	1	1	1	1	0	0	0	0	−1	−1	−1	1	1	1	−1	−1	−1	−1	−1	2	2	3	3	3	0	0
χ_6	5	5	5	5	2	2	1	1	1	1	1	0	0	0	0	3	3	3	1	1	1	0	0	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_7	6	6	6	−3	0	0	−2	−2	−2	−2	1	1	1	A	$/A$	0	0	0	0	0	0	0	0	−1	2	2	0	0	0	0	0	0	0
χ_8	6	6	6	−3	0	0	−2	−2	−2	−2	1	1	1	$/A$	A	0	0	0	0	0	0	0	0	−1	2	2	0	0	0	0	0	0	0
χ_9	9	9	9	9	0	0	1	1	1	1	1	−1	−1	−1	−1	−3	−3	−3	1	1	1	0	0	1	1	1	0	0	−3	−3	−3	0	0
χ_{10}	9	9	9	9	0	0	1	1	1	1	1	−1	−1	−1	−1	3	3	3	−1	−1	−1	0	0	1	1	1	0	0	3	3	3	0	0
χ_{11}	10	10	10	10	1	1	−2	−2	−2	−2	−2	0	0	0	0	−2	−2	−2	0	0	0	1	1	0	0	0	1	1	2	2	2	−1	−1
χ_{12}	10	10	10	10	1	1	−2	−2	−2	−2	−2	0	0	0	0	2	2	2	0	0	0	−1	−1	0	0	0	1	1	−2	−2	−2	1	1
χ_{13}	12	12	12	−6	0	0	4	4	4	4	−2	2	2	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	16	16	16	16	−2	−2	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	−2	−2	0	0	0	0	0
χ_{15}	18	2	−6	0	3	−1	2	6	−2	−2	0	−1	3	0	0	0	−4	4	0	2	−2	1	−1	0	0	0	0	0	0	0	0	0	0
χ_{16}	18	2	−6	0	3	−1	2	6	−2	−2	0	−1	3	0	0	0	4	−4	0	−2	2	−1	1	0	0	0	0	0	0	0	0	0	0
χ_{17}	18	18	18	−9	0	0	2	2	2	2	−1	−2	−2	1	1	0	0	0	0	0	0	0	0	−1	2	2	0	0	0	0	0	0	0
χ_{18}	30	30	30	−15	0	0	−2	−2	−2	−2	1	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	−2	0	0	0	0	0	0	0
χ_{19}	45	−3	5	0	0	0	−3	9	1	1	0	0	0	0	0	−1	3	3	−1	1	1	0	0	0	3	−1	−1	3	−1	−1	7	1	−1
χ_{20}	45	−3	5	0	0	0	−3	9	1	1	0	0	0	0	0	1	−3	−3	1	−1	−1	0	0	0	3	−1	−1	3	1	1	−7	−1	1
χ_{21}	45	−3	5	0	0	0	1	−3	−3	5	0	0	0	0	0	1	−3	−3	−1	1	1	0	0	0	−3	1	−1	3	1	−3	5	−1	1
χ_{22}	45	−3	5	0	0	0	1	−3	−3	5	0	0	0	0	0	−1	3	3	1	−1	−1	0	0	0	−3	1	−1	3	−1	3	−5	1	−1
χ_{23}	72	8	−24	0	3	−1	0	0	0	0	0	1	−3	0	0	0	−8	8	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	0
χ_{24}	72	8	−24	0	3	−1	0	0	0	0	0	1	−3	0	0	0	8	−8	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0
χ_{25}	90	−6	10	0	0	0	−2	6	−2	6	0	0	0	0	0	−2	6	6	0	0	0	0	0	0	0	0	1	−3	−2	2	2	−1	1
χ_{26}	90	−6	10	0	0	0	−2	6	−2	6	0	0	0	0	0	2	−6	−6	0	0	0	0	0	0	0	0	1	−3	2	−2	−2	1	−1
χ_{27}	90	10	−30	0	−3	1	2	6	−2	−2	0	0	0	0	0	0	−4	4	0	−2	2	1	−1	0	0	0	0	0	0	0	0	0	0
χ_{28}	90	10	−30	0	−3	1	2	6	−2	−2	0	0	0	0	0	0	4	−4	0	2	−2	−1	1	0	0	0	0	0	0	0	0	0	0
χ_{29}	108	12	−36	0	0	0	−4	−12	4	4	0	−1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	135	−9	15	0	0	0	−1	3	3	−5	0	0	0	0	0	−1	3	3	−1	1	1	0	0	0	−3	1	0	0	3	−1	−9	0	0
χ_{31}	135	−9	15	0	0	0	−1	3	3	−5	0	0	0	0	0	1	−3	−3	1	−1	−1	0	0	0	−3	1	0	0	−3	1	9	0	0
χ_{32}	135	−9	15	0	0	0	3	−9	−1	−1	0	0	0	0	0	1	−3	−3	−1	1	1	0	0	0	3	−1	0	0	−3	5	−3	0	0
χ_{33}	135	−9	15	0	0	0	3	−9	−1	−1	0	0	0	0	0	−1	3	3	1	−1	−1	0	0	0	3	−1	0	0	3	−5	3	0	0

Table 7: Fischer Matrices of $2^{11}{:}M_{24}$

Fischer Matrices of $2^{11}{:}M_{24}$	Fischer Matrices of $2^{11}{:}M_{24}$
$M(1A)=\begin{pmatrix}1&1&1\\276&20&-12\\1771&-21&11\end{pmatrix}$	$M(2A)=\begin{pmatrix}1&1&1&1&1\\12&0&-4&-12&4\\1&-1&1&1&1\\30&0&-2&30&-2\\20&0&4&-20&-4\end{pmatrix}$
$M(2B)=\begin{pmatrix}1&1&1&1&1&1\\28&4&-4&-4&-4&-28\\8&0&0&8&-8&-8\\7&-1&-1&7&7&7\\28&-4&4&-4&-4&-28\\56&0&0&-8&8&56\end{pmatrix}$	$M(3A)=\begin{pmatrix}1&1&1&1\\15&-1&3&-5\\1&1&-1&-1\\15&-1&-3&5\end{pmatrix}$
$M(3B)=\begin{pmatrix}1&1\\7&-1\end{pmatrix}$	$M(4A)=\begin{pmatrix}1&1&1&1&1&1&1\\2&0&-2&2&-2&2&-2\\2&0&-2&-2&2&2&2\\4&0&0&0&-4&-4&4\\1&-1&1&1&1&1&1\\2&0&2&-2&-2&2&-2\\4&0&0&0&4&-4&-4\end{pmatrix}$
$M(4B)=\begin{pmatrix}1&1&1&1\\3&-3&1&-1\\3&3&-1&-1\\1&-1&-1&1\end{pmatrix}$	$M(4C)=\begin{pmatrix}1&1&1&1&1&1\\4&0&-4&0&4&-4\\3&-1&3&-1&3&3\\1&-1&1&1&-1&-1\\4&0&-4&0&-4&4\\3&1&3&-1&-3&-3\end{pmatrix}$
$M(5A)=\begin{pmatrix}1&1&1\\6&0&-2\\1&-1&1\end{pmatrix}$	$M(6A)=\begin{pmatrix}1&1&1\\1&-1&1\\2&0&-2\end{pmatrix}$
$M(6B)=\begin{pmatrix}1&1&1&1&1&1\\1&1&-1&-1&-1&1\\2&-2&-2&2&0&0\\1&1&-1&-1&1&-1\\1&1&1&1&-1&-1\\2&-2&2&-2&0&0\end{pmatrix}$	$M(7A)=\begin{pmatrix}1&1\\3&-1\end{pmatrix}$
$M(7B)=\begin{pmatrix}1&1\\3&-1\end{pmatrix}$	$M(8A)=\begin{pmatrix}1&1&1&1\\1&-1&-1&1\\1&-1&1&-1\\1&1&-1&-1\end{pmatrix}$
$M(10A)=\begin{pmatrix}1&1&1\\2&-2&0\\1&1&-1\end{pmatrix}$	$M(11A)=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$
$M(12A)=\begin{pmatrix}1&1&1&1\\1&-1&-1&1\\1&-1&1&-1\\1&1&-1&-1\end{pmatrix}$	$M(12B)=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$
$M(14A)=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$	$M(14B)=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$
$M(15A)=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$	$M(15B)=\begin{pmatrix}1&1\\1&-1\end{pmatrix}$
$M(21A)=(1)$	$M(21B)=(1)$
$M(23A)=(1)$	$M(23B)=(1)$

Table 8: The Character Table of $\bar{G} = 2^{11}{:}M_{24}$

$[g]_{\bar{G}}$	$1A$	$2A$	$2B$	$2E$	$2F$	$4E$	$4F$	$4G$	$2C$	$2D$	$4A$	$4B$	$4C$	$4D$
$\lvert C_{\bar{G}}(g)\rvert$	501397585920	660602880	389283840	491520	15360	32768	32768	491520	344064	6144	6144	344064	49152	49152
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	23	23	23	−1	−1	−1	−1	−1	7	7	7	7	7	7
χ_3	45	45	45	5	5	5	5	5	−3	−3	−3	−3	−3	−3
χ_4	45	45	45	5	5	5	5	5	−3	−3	−3	−3	−3	−3
χ_5	231	231	231	−9	−9	−9	−9	−9	7	7	7	7	7	7
χ_6	231	231	231	−9	−9	−9	−9	−9	7	7	7	7	7	7
χ_7	252	252	252	12	12	12	12	12	28	28	28	28	28	28
χ_8	253	253	253	−11	−11	−11	−11	−11	13	13	13	13	13	13
χ_9	483	483	483	3	3	3	3	3	35	35	35	35	35	35
χ_{10}	770	770	770	10	10	10	10	10	−14	−14	−14	−14	−14	−14
χ_{11}	770	770	770	10	10	10	10	10	−14	−14	−14	−14	−14	−14
χ_{12}	990	990	990	−10	−10	−10	−10	−10	−18	−18	−18	−18	−18	−18
χ_{13}	990	990	990	−10	−10	−10	−10	−10	−18	−18	−18	−18	−18	−18
χ_{14}	1035	1035	1035	35	35	35	35	35	27	27	27	27	27	27
χ_{15}	1035	1035	1035	−5	−5	−5	−5	−5	−21	−21	−21	−21	−21	−21
χ_{16}	1035	1035	1035	−5	−5	−5	−5	−5	−21	−21	−21	−21	−21	−21
χ_{17}	1265	1265	1265	−15	−15	−15	−15	−15	49	49	49	49	49	49
χ_{18}	1771	1771	1771	11	11	11	11	11	−21	−21	−21	−21	−21	−21
χ_{19}	2024	2024	2024	24	24	24	24	24	8	8	8	8	8	8
χ_{20}	2277	2277	2277	−19	−19	−19	−19	−19	21	21	21	21	21	21
χ_{21}	3312	3312	3312	16	16	16	16	16	48	48	48	48	48	48
χ_{22}	3520	3520	3520	0	0	0	0	0	64	64	64	64	64	64
χ_{23}	5313	5313	5313	9	9	9	9	9	49	49	49	49	49	49
χ_{24}	5544	5544	5544	24	24	24	24	24	−56	−56	−56	−56	−56	−56
χ_{25}	5796	5796	5796	36	36	36	36	36	−28	−28	−28	−28	−28	−28
χ_{26}	10395	10395	10395	−45	−45	−45	−45	−45	−21	−21	−21	−21	−21	−21
χ_{27}	276	20	−12	12	−12	−4	4	0	36	4	20	−12	−4	4
χ_{28}	276	20	−12	−12	12	4	−4	0	20	−12	36	4	−4	4
χ_{29}	5796	420	−252	−12	12	4	−4	0	196	36	84	−76	−20	20
χ_{30}	5796	420	−252	12	−12	−4	4	0	84	−76	196	36	−20	20
χ_{31}	12420	900	−540	−60	60	20	−20	0	−60	36	−108	−12	12	−12
χ_{32}	12420	900	−540	60	−60	−20	20	0	−108	−12	−60	36	12	−12
χ_{33}	12420	900	−540	−60	60	20	−20	0	−60	36	−108	−12	12	−12
χ_{34}	12420	900	−540	60	−60	−20	20	0	−108	−12	−60	36	12	−12
χ_{35}	15180	1100	−660	60	−60	−20	20	0	300	76	92	−132	−28	28
χ_{36}	15180	1100	−660	−60	60	20	−20	0	92	−132	300	76	−28	28
χ_{37}	27324	1980	−1188	−12	12	4	−4	0	204	108	−36	−132	−12	12
χ_{38}	27324	1980	−1188	12	−12	−4	4	0	−36	−132	204	108	−12	12
χ_{39}	42504	3080	−1848	72	−72	−24	24	0	392	72	168	−152	−40	40
χ_{40}	42504	3080	−1848	−72	72	24	−24	0	168	−152	392	72	−40	40
χ_{41}	57960	4200	−2520	−120	120	40	−40	0	168	104	−56	−120	−8	8

Table 8: The Character Table of $\bar{G} = 2^{11}{:}M_{24}$ (Continued)

$[g]_{\bar{G}}$	$1A$	$2A$	$2B$	$2E$	$2F$	$4E$	$4F$	$4G$	$2C$	$2D$	$4A$	$4B$	$4C$	$4D$
$\lvert C_{\bar{G}}(g)\rvert$	501397585920	660602880	389283840	491520	15360	32768	32768	491520	344064	6144	6144	344064	49152	49152
χ_{42}	57960	4200	-2520	120	-120	-40	40	0	-56	-120	168	104	-8	8
χ_{43}	63756	4620	-2772	-108	108	36	-36	0	252	28	140	-84	-28	28
χ_{44}	63756	4620	-2772	108	-108	-36	36	0	140	-84	252	28	-28	28
χ_{45}	154560	11200	-6720	0	0	0	0	0	-448	64	-448	64	64	-64
χ_{46}	106260	7700	-4620	60	-60	-20	20	0	196	164	-140	-172	-4	4
χ_{47}	106260	7700	-4620	-60	60	20	-20	0	-140	-172	196	164	-4	4
χ_{48}	1771	-21	11	51	11	3	-5	-1	91	-5	-21	11	3	-5
χ_{49}	1771	-21	11	11	51	-5	3	-1	-21	11	91	-5	3	-5
χ_{50}	8855	-105	55	15	55	-1	7	-5	231	7	-105	55	-1	-9
χ_{51}	8855	-105	55	55	15	7	-1	-5	-105	55	231	7	-1	-9
χ_{52}	8855	-105	55	-25	95	-9	15	-5	119	23	7	39	-1	-9
χ_{53}	8855	-105	55	95	-25	15	-9	-5	7	39	119	23	-1	-9
χ_{54}	15939	-189	99	-21	99	-5	19	-9	-77	83	259	35	-5	-13
χ_{55}	15939	-189	99	99	-21	19	-5	-9	259	35	-77	83	-5	-13
χ_{56}	17710	-210	110	-10	-90	22	6	-10	-98	94	126	62	-18	-2
χ_{57}	17710	-210	110	-90	-10	6	22	-10	126	62	-98	94	-18	-2
χ_{58}	28336	-336	176	16	16	16	16	-16	112	112	112	112	-16	-16
χ_{59}	10626	-126	66	-54	-54	10	10	-6	-14	50	-14	50	-14	2
χ_{60}	10626	-126	66	-54	-54	10	10	-6	-14	50	-14	50	-14	2
χ_{61}	21252	-252	132	132	132	4	4	-12	196	68	196	68	4	-28
χ_{62}	31878	-378	198	78	78	14	14	-18	182	118	182	118	-10	-26
χ_{63}	53130	-630	330	-30	-30	34	34	-30	154	218	154	218	-38	-22
χ_{64}	31878	-378	198	-66	-66	-2	-2	6	406	-42	-42	22	22	-26
χ_{65}	31878	-378	198	-66	-66	-2	-2	6	-42	22	406	-42	22	-26
χ_{66}	79695	-945	495	255	55	15	-25	-5	-273	15	63	-33	-9	15
χ_{67}	79695	-945	495	55	255	-25	15	-5	63	-33	-273	15	-9	15
χ_{68}	79695	-945	495	175	-105	31	-25	-5	399	-81	63	-33	39	-33
χ_{69}	79695	-945	495	-105	175	-25	31	-5	63	-33	399	-81	39	-33
χ_{70}	127512	-1512	792	-24	-24	-24	-24	24	-392	120	504	-8	-8	-8
χ_{71}	127512	-1512	792	-24	-24	-24	-24	24	504	-8	-392	120	-8	-8
χ_{72}	159390	-1890	990	-90	-90	-26	-26	30	462	14	14	78	14	-34
χ_{73}	159390	-1890	990	-90	-90	-26	-26	30	14	78	462	14	14	-34
χ_{74}	159390	-1890	990	230	150	6	-10	-10	462	-114	-210	-18	30	-18
χ_{75}	159390	-1890	990	150	230	-10	6	-10	-210	-18	462	-114	30	-18
χ_{76}	191268	-2268	1188	84	84	-44	-44	36	-252	132	-252	132	-60	36
χ_{77}	239085	-2835	1485	-315	45	-11	61	-15	189	-99	-147	-51	21	-3
χ_{78}	239085	-2835	1485	45	-315	61	-11	-15	-147	-51	189	-99	21	-3
χ_{79}	239085	-2835	1485	45	-75	29	5	-15	-147	-51	-483	-3	-27	45
χ_{80}	239085	-2835	1485	-75	45	5	29	-15	-483	-3	-147	-51	-27	45

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$3A$	$6A$	$6B$	$6C$	$6B$	$6D$	$4K$	$4L$	$8D$	$8E$	$4M$	$8F$	$8G$	$4N$	$8H$	$4O$	$8I$
$\lvert C_{\bar{G}}(g)\rvert$	34560	2304	3456	5760	4032	576	2048	1024	256	2048	1024	2048	2048	768	256	256	768
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	5	5	5	5	−1	−1	3	3	3	3	3	3	3	−1	−1	−1	−1
χ_3	0	0	0	0	3	3	1	1	1	1	1	1	1	1	1	1	1
χ_4	0	0	0	0	3	3	1	1	1	1	1	1	1	1	1	1	1
χ_5	−3	−3	−3	−3	0	0	−1	−1	−1	−1	−1	−1	−1	3	3	3	3
χ_6	−3	−3	−3	−3	0	0	−1	−1	−1	−1	−1	−1	−1	3	3	3	3
χ_7	9	9	9	9	0	0	4	4	4	4	4	4	4	0	0	0	0
χ_8	10	10	10	10	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_9	6	6	6	6	0	0	3	3	3	3	3	3	3	3	3	3	3
χ_{10}	5	5	5	5	−7	−7	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
χ_{11}	5	5	5	5	−7	−7	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
χ_{12}	0	0	0	0	3	3	2	2	2	2	2	2	2	−2	−2	−2	−2
χ_{13}	0	0	0	0	3	3	2	2	2	2	2	2	2	−2	−2	−2	−2
χ_{14}	0	0	0	0	6	6	−1	−1	−1	−1	−1	−1	−1	3	3	3	3
χ_{15}	0	0	0	0	−3	−3	3	3	3	3	3	3	3	−1	−1	−1	−1
χ_{16}	0	0	0	0	−3	−3	3	3	3	3	3	3	3	−1	−1	−1	−1
χ_{17}	5	5	5	5	8	8	1	1	1	1	1	1	1	−3	−3	−3	−3
χ_{18}	16	16	16	16	7	7	−5	−5	−5	−5	−5	−5	−5	−1	−1	−1	−1
χ_{19}	−1	−1	−1	−1	8	8	−5	−5	−5	−5	−5	−5	−5	−1	−1	−1	−1
χ_{20}	0	0	0	0	6	6	1	1	1	1	1	1	1	−3	−3	−3	−3
χ_{21}	0	0	0	0	−6	−6	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	10	10	10	10	−8	−8	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	−15	−15	−15	−15	0	0	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{24}	9	9	9	9	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	−9	−9	−9	−9	0	0	4	4	4	4	4	4	4	0	0	0	0
χ_{26}	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1	3	3	3	3
χ_{27}	15	−1	5	−3	0	0	8	0	4	−4	−4	0	0	0	0	0	0
χ_{28}	15	−1	5	−3	0	0	4	−4	8	0	0	−4	0	0	0	0	0
χ_{29}	45	−3	15	−9	0	0	12	4	0	−8	−8	4	0	0	0	0	0
χ_{30}	45	−3	15	−9	0	0	0	−8	12	4	4	−8	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	4	−4	8	0	0	−4	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	8	0	4	−4	−4	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	4	−4	8	0	0	−4	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	8	0	4	−4	−4	0	0	0	0	0	0
χ_{35}	15	−1	5	−3	0	0	4	12	0	8	−8	−4	0	0	0	0	0
χ_{36}	15	−1	5	−3	0	0	0	8	4	12	−4	−8	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	8	4	12	−4	−8	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	4	12	0	8	−8	−4	0	0	0	0	0
χ_{39}	15	−1	5	−3	0	0	8	−8	0	−16	0	8	0	0	0	0	0
χ_{40}	15	−1	5	−3	0	0	0	−16	8	−8	8	0	0	0	0	0	0
χ_{41}	45	−3	15	−9	0	0	−8	8	−16	0	0	8	0	0	0	0	0

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$3A$	$6A$	$6B$	$6C$	$6B$	$6D$	$4K$	$4L$	$8D$	$8E$	$4M$	$8F$	$8G$	$4N$	$8H$	$4O$	$8I$
$\lvert C_{\bar{G}}(g)\rvert$	34560	2304	3456	5760	4032	576	2048	1024	256	2048	1024	2048	2048	768	256	256	768
χ_{42}	45	−3	15	−9	0	0	−16	0	−8	8	8	0	0	0	0	0	0
χ_{43}	−45	3	−15	9	0	0	−8	0	−4	4	4	0	0	0	0	0	0
χ_{44}	−45	3	−15	9	0	0	−4	4	−8	0	0	4	0	0	0	0	0
χ_{45}	30	−2	10	−6	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	−30	2	−10	6	0	0	0	−8	12	4	4	−8	0	0	0	0	0
χ_{47}	−30	2	−10	6	0	0	12	4	0	−8	−8	4	0	0	0	0	0
χ_{48}	16	0	−6	2	7	−1	7	−1	−5	3	3	−1	−1	7	−1	−1	−1
χ_{49}	16	0	−6	2	7	−1	−5	3	7	−1	−1	3	−1	−1	−1	7	−1
χ_{50}	35	3	−15	1	−7	1	11	3	−9	−1	7	−5	−1	−1	−1	−5	3
χ_{51}	35	3	−15	1	−7	1	−9	−1	11	3	−5	7	−1	−5	3	−1	−1
χ_{52}	−10	6	0	−8	14	−2	−1	7	3	−5	3	−1	−1	−9	−1	3	3
χ_{53}	−10	6	0	−8	14	−2	3	−5	−1	7	−1	3	−1	3	3	−9	−1
χ_{54}	9	9	−9	−9	0	0	−1	−9	3	11	−5	7	−1	3	−5	3	3
χ_{55}	9	9	−9	−9	0	0	3	11	−1	−9	7	−5	−1	3	3	3	−5
χ_{56}	25	9	−15	−7	7	−1	−6	−6	2	2	−6	2	2	2	2	−2	−2
χ_{57}	25	9	−15	−7	7	−1	2	2	−6	−6	2	−6	2	−2	−2	2	2
χ_{58}	−14	18	−6	−22	−14	2	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	−3	−3	3	3	0	0	−2	−2	−2	−2	−2	−2	2	6	−2	6	−2
χ_{60}	−3	−3	3	3	0	0	−2	−2	−2	−2	−2	−2	2	6	−2	6	−2
χ_{61}	−6	−6	6	6	0	0	4	4	4	4	4	4	−4	0	0	0	0
χ_{62}	−9	−9	9	9	0	0	2	2	2	2	2	2	−2	6	−2	6	−2
χ_{63}	−15	−15	15	15	0	0	−2	−2	−2	−2	−2	−2	2	−6	2	−6	2
χ_{64}	45	−3	−15	9	0	0	10	−6	−6	10	2	2	−2	−6	2	6	−2
χ_{65}	45	−3	−15	9	0	0	−6	10	10	−6	2	2	−2	6	−2	−6	2
χ_{66}	0	0	0	0	21	−3	7	−1	−5	3	3	−1	−1	7	−1	−1	−1
χ_{67}	15	0	0	0	0	21	−5	3	7	−1	−1	3	−1	−1	−1	7	−1
χ_{68}	0	0	0	0	21	−3	−1	−9	−5	3	−5	−1	3	−1	−1	−5	3
χ_{69}	0	0	0	0	21	−3	−5	3	−1	−9	−1	−5	3	−5	3	−1	−1
χ_{70}	45	−3	−15	9	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	45	−3	−15	9	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	−45	3	15	−9	0	0	−6	10	10	−6	2	2	−2	6	−2	−6	2
χ_{73}	−45	3	15	−9	0	0	10	−6	−6	10	2	2	−2	−6	2	6	−2
χ_{74}	0	0	0	0	−21	3	−6	−6	2	2	−6	2	2	−2	−2	2	2
χ_{75}	0	0	0	0	−21	3	2	2	−6	−6	2	−6	2	2	2	−2	−2
χ_{76}	0	0	0	0	0	0	−4	−4	−4	−4	−4	−4	4	0	0	0	0
χ_{77}	0	0	0	0	0	0	1	−7	−3	5	−3	1	1	9	1	−3	−3
χ_{78}	0	0	0	0	0	0	−3	5	1	−7	1	−3	1	−3	−3	9	1
χ_{79}	0	0	0	0	0	0	−3	5	9	1	1	5	−3	−3	5	−3	−3
χ_{80}	0	0	0	0	0	0	9	1	−3	5	5	1	−3	−3	−3	−3	5

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$4H$	$4I$	$8A$	$8B$	$4J$	$8C$	$5A$	$10A$	$10B$	$6E$	$6F$	$12A$	$12B$	$12C$	$12D$	$6G$	$6H$	$12E$	$7A$	$14A$
$\|C_{\bar{G}}(g)\|$	6144	1024	6144	1024	6144	6144	2480	160	120	192	96	192	96	192	192	96	48	96	168	56
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	−1	−1	−1	−1	−1	−1	3	3	3	1	1	1	1	1	1	−1	−1	−1	2	2
χ_3	−3	−3	−3	−3	−3	−3	0	0	0	0	0	0	0	0	0	−1	−1	−1	A	A
χ_4	−3	−3	−3	−3	−3	−3	0	0	0	0	0	0	0	0	0	−1	−1	−1	$/A$	$/A$
χ_5	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
χ_6	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
χ_7	4	4	4	4	4	4	2	2	2	1	1	1	1	1	1	0	0	0	0	0
χ_8	−3	−3	−3	−3	−3	−3	3	3	3	−2	−2	−2	−2	−2	−2	1	1	1	1	1
χ_9	3	3	3	3	3	3	−2	−2	−2	2	2	2	2	2	2	0	0	0	0	0
χ_{10}	2	2	2	2	2	2	0	0	0	1	1	1	1	1	1	1	1	1	0	0
χ_{11}	2	2	2	2	2	2	0	0	0	1	1	1	1	1	1	1	1	1	0	0
χ_{12}	6	6	6	6	6	6	0	0	0	0	0	0	0	0	0	−1	−1	−1	A	A
χ_{13}	6	6	6	6	6	6	0	0	0	0	0	0	0	0	0	−1	−1	−1	$/A$	$/A$
χ_{14}	3	3	3	3	3	3	0	0	0	0	0	0	0	0	0	2	2	2	−1	−1
χ_{15}	3	3	3	3	3	3	0	0	0	0	0	0	0	0	0	1	1	1	B	B
χ_{16}	3	3	3	3	3	3	0	0	0	0	0	0	0	0	0	1	1	1	$/B$	$/B$
χ_{17}	−7	−7	−7	−7	−7	−7	0	0	0	1	1	1	1	1	1	0	0	0	−2	−2
χ_{18}	3	3	3	3	3	3	1	1	1	0	0	0	0	0	0	−1	−1	−1	0	0
χ_{19}	8	8	8	8	8	8	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	1	1
χ_{20}	−3	−3	−3	−3	−3	−3	−3	−3	−3	0	0	0	0	0	0	2	2	2	2	2
χ_{21}	0	0	0	0	0	0	−3	−3	−3	0	0	0	0	0	0	−2	−2	−2	1	1
χ_{22}	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2	0	0	0	−1	−1
χ_{23}	1	1	1	1	1	1	3	3	3	1	1	1	1	1	1	0	0	0	0	0
χ_{24}	−8	−8	−8	−8	−8	−8	−1	−1	−1	1	1	1	1	1	1	0	0	0	0	0
χ_{25}	−4	−4	−4	−4	−4	−4	1	1	1	−1	−1	−1	−1	−1	−1	0	0	0	0	0
χ_{26}	3	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	4	−4	4	−4	0	0	6	0	−2	3	1	−1	−3	−1	1	0	0	0	3	−1
χ_{28}	−4	4	−4	4	0	0	6	0	−2	−1	−3	3	1	−1	1	0	0	0	3	−1
χ_{29}	−4	4	−4	4	0	0	6	0	−2	1	3	−3	−1	1	−1	0	0	0	0	0
χ_{30}	4	−4	4	−4	0	0	6	0	−2	−3	−1	1	3	1	−1	0	0	0	0	0
χ_{31}	12	−12	12	−12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	$-A$
χ_{32}	−12	12	−12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$/C$	$-/A$
χ_{33}	12	−12	12	−12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$/C$	$-/A$
χ_{34}	−12	12	−12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$/C$	$-/A$
χ_{35}	4	−4	4	−4	0	0	0	0	0	3	1	−1	−3	−1	1	0	0	0	−3	1
χ_{36}	−4	4	−4	4	0	0	0	0	0	−1	−3	3	1	−1	1	0	0	0	−3	1
χ_{37}	12	−12	12	−12	0	0	−6	0	2	0	0	0	0	0	0	0	0	0	3	−1
χ_{38}	−12	12	−12	12	0	0	−6	0	2	0	0	0	0	0	0	0	0	0	3	−1
χ_{39}	8	−8	8	−8	0	0	−6	0	2	−1	−3	3	1	−1	1	0	0	0	0	0
χ_{40}	−8	8	−8	8	0	0	−6	0	2	3	1	−1	−3	−1	1	0	0	0	0	0
χ_{41}	−8	8	−8	8	0	0	0	0	0	−3	−1	1	3	1	−1	0	0	0	0	0

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$4H$	$4I$	$8A$	$8B$	$4J$	$8C$	$5A$	$10A$	$10B$	$6E$	$6F$	$12A$	$12B$	$12C$	$12D$	$6G$	$6H$	$12E$	$7A$	$14A$
$\lvert C_{\bar{G}}(g)\rvert$	6144	1024	6144	1024	6144	6144	2480	160	120	192	96	192	96	192	192	96	48	96	168	56
χ_{42}	8	−8	8	−8	0	0	0	0	0	1	3	−3	−1	1	−1	0	0	0	0	0
χ_{43}	−4	4	−4	4	0	0	6	0	−2	3	1	−1	−3	−1	1	0	0	0	0	0
χ_{44}	4	−4	4	−4	0	0	6	0	−2	−1	−3	3	1	−1	1	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	2	−2	2	−2	−2	2	0	0	0	0	0
χ_{46}	−12	12	−12	12	0	0	0	0	0	−2	2	−2	2	2	−2	0	0	0	0	0
χ_{47}	12	−12	12	−12	0	0	0	0	0	−2	2	−2	2	2	−2	0	0	0	0	0
χ_{48}	11	3	−5	3	−1	−1	1	−1	1	4	−2	0	2	0	−2	3	−1	−1	0	0
χ_{49}	3	−5	3	11	−1	−1	1	−1	1	0	2	4	−2	0	−2	−1	3	−1	0	0
χ_{50}	−1	7	7	−1	3	−5	0	0	0	3	1	3	1	−1	−3	−3	1	1	0	0
χ_{51}	−1	7	7	−1	−5	3	0	0	0	3	1	3	1	−1	−3	1	−3	1	0	0
χ_{52}	−9	−1	15	7	3	−5	0	0	0	2	−4	−2	0	2	0	2	2	−2	0	0
χ_{53}	7	15	−1	−9	−5	3	0	0	0	−2	0	2	−4	2	0	2	2	−2	0	0
χ_{54}	−5	−13	11	19	−1	−1	−1	1	−1	1	−1	1	−1	1	−1	0	0	0	0	0
χ_{55}	19	11	−13	−5	−1	−1	−1	1	−1	1	−1	1	−1	1	−1	0	0	0	0	0
χ_{56}	−2	−2	−10	−10	−2	6	0	0	0	1	1	−3	5	−3	1	−1	3	−1	0	0
χ_{57}	−10	−10	−2	−2	6	−2	0	0	0	−3	5	1	1	−3	1	3	−1	−1	0	0
χ_{58}	0	0	0	0	0	0	1	−1	1	−2	−2	−2	−2	2	2	−2	−2	2	0	0
χ_{59}	2	−14	−14	2	2	2	1	−1	1	1	−1	1	−1	1	−1	0	0	0	0	0
χ_{60}	2	−14	−14	2	2	2	1	−1	1	1	−1	1	−1	1	−1	0	0	0	0	0
χ_{61}	12	12	12	12	−4	−4	2	−2	2	−2	2	−2	2	−2	2	0	0	0	0	0
χ_{62}	14	−2	−2	14	−2	−2	2	−2	2	−1	1	−1	1	−1	1	0	0	0	0	0
χ_{63}	−14	2	2	−14	2	2	0	0	0	1	−1	1	−1	1	−1	0	0	0	0	0
χ_{64}	−10	−10	−2	−2	−2	6	3	−3	3	1	−3	−3	1	1	1	0	0	0	0	0
χ_{65}	−2	−2	−10	−10	6	−2	3	−3	3	−3	1	1	−3	1	1	0	0	0	0	0
χ_{66}	−33	−9	15	−9	3	3	0	0	0	0	0	0	0	0	0	−3	1	1	0	0
χ_{67}	−9	15	−9	−33	3	3	0	0	0	0	0	0	0	0	0	1	−3	1	0	0
χ_{68}	15	−9	−9	15	3	−5	0	0	0	0	0	0	0	0	0	1	−3	1	0	0
χ_{69}	15	−9	−9	15	−5	3	0	0	0	1	−3	−3	1	1	0	−3	1	1	0	0
χ_{70}	8	8	−8	−8	8	−8	−3	3	−3	1	−3	−3	1	1	1	0	0	0	0	0
χ_{71}	−8	−8	8	8	−8	8	−3	3	−3	−3	1	1	−3	1	1	0	0	0	0	0
χ_{72}	−10	−10	−2	−2	−2	6	0	0	0	3	−1	−1	3	−1	−1	0	0	0	0	0
χ_{73}	−2	−2	−10	−10	6	−2	0	0	0	−1	3	3	−1	−1	−1	0	0	0	0	0
χ_{74}	6	6	−18	−18	6	−2	0	0	0	0	0	0	0	0	0	−1	3	−1	0	0
χ_{75}	−18	−18	6	6	−2	6	0	0	0	0	0	0	0	0	0	3	−1	−1	0	0
χ_{76}	12	12	12	12	−4	−4	3	−3	3	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	−3	21	21	−3	1	−7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	−3	21	21	−3	−7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	−3	−27	−3	21	9	−7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	21	−3	−27	−3	−7	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$7B$	$14B$	$8J$	$8K$	$16A$	$16B$	$10C$	$10D$	$20A$	$11A$	$22A$	$12F$	$24A$	$12G$	$24B$	$12H$	$12I$	$14C$	$28A$
$\lvert C_{\bar{G}}(g)\rvert$	168	56	256	128	32	256	80	40	80	22	22	48	48	48	48	24	24	28	28
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	2	2	1	1	1	1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	0	0
χ_3	$/A$	$/A$	-1	-1	-1	-1	0	0	0	1	1	0	0	0	0	1	1	$-A$	$-A$
χ_4	A	A	-1	-1	-1	-1	0	0	0	1	1	0	0	0	0	1	1	$-/A$	$-/A$
χ_5	0	0	-1	-1	-1	-1	1	1	1	0	0	-1	-1	-1	-1	0	0	0	0
χ_6	0	0	-1	-1	-1	-1	1	1	1	0	0	-1	-1	-1	-1	0	0	0	0
χ_7	0	0	0	0	0	0	2	2	2	-1	-1	1	1	1	1	0	0	0	0
χ_8	1	1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	1	1	-1	-1
χ_9	0	0	-1	-1	-1	-1	-2	-2	-2	-1	-1	0	0	0	0	0	0	0	0
χ_{10}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	0	0
χ_{11}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	0	0
χ_{12}	$/A$	$/A$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	A	A
χ_{13}	A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	$/A$	$/A$
χ_{14}	-1	-1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	0	-1	-1
χ_{15}	$/B$	$/B$	-1	-1	-1	-1	0	0	0	1	1	0	0	0	0	-1	-1	0	0
χ_{16}	B	B	-1	-1	-1	-1	0	0	0	1	1	0	0	0	0	-1	-1	0	0
χ_{17}	-2	-2	1	1	1	1	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0
χ_{18}	0	0	-1	-1	-1	-1	1	1	1	0	0	0	0	0	0	-1	-1	0	0
χ_{19}	1	1	0	0	0	0	-1	-1	-1	0	0	-1	-1	-1	-1	0	0	1	1
χ_{20}	2	2	-1	-1	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{21}	1	1	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	-1	-1
χ_{22}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
χ_{23}	0	0	-1	-1	-1	-1	-1	-1	-1	0	0	1	1	1	1	0	0	0	0
χ_{24}	0	0	0	0	0	0	-1	-1	-1	0	0	1	1	1	1	0	0	0	0
χ_{25}	0	0	0	0	0	0	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0
χ_{26}	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	3	-1	2	-2	0	0	2	-2	0	1	-1	1	-1	-1	1	0	0	1	-1
χ_{28}	3	-1	0	0	2	-2	-2	2	0	1	-1	-1	1	1	-1	0	0	-1	1
χ_{29}	0	0	0	0	-2	2	-2	2	0	-1	1	-1	1	1	-1	0	0	0	0
χ_{30}	0	0	-2	2	0	0	2	-2	0	-1	1	1	-1	-1	1	0	0	0	0
χ_{31}	$/C$	$-/A$	0	0	-2	2	0	0	0	1	-1	0	0	0	0	0	0	A	$-A$
χ_{32}	C	$-A$	0	0	-2	2	0	0	0	1	-1	0	0	0	0	0	0	$-A$	A
χ_{33}	C	$-A$	-2	2	0	0	0	0	0	1	-1	0	0	0	0	0	0	$/A$	$-/A$
χ_{34}	C	$-A$	-2	2	0	0	0	0	0	1	-1	0	0	0	0	0	0	$-/A$	$/A$
χ_{35}	-3	1	0	0	2	-2	0	0	0	0	0	1	-1	-1	1	0	0	-1	1
χ_{36}	-3	1	2	-2	0	0	0	0	0	0	0	-1	1	1	-1	0	0	1	-1
χ_{37}	3	-1	-2	2	0	0	-2	2	0	0	0	0	0	0	0	0	0	1	-1
χ_{38}	3	-1	0	0	-2	2	2	-2	0	0	0	0	0	0	0	0	0	-1	1
χ_{39}	0	0	0	0	0	0	2	-2	0	0	0	-1	1	1	-1	0	0	0	0
χ_{40}	0	0	0	0	0	0	-2	2	0	0	0	1	-1	-1	1	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	1	0	0	0	0

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$7B$	$14B$	$8J$	$8K$	$16A$	$16B$	$10C$	$10D$	$20A$	$11A$	$22A$	$12F$	$24A$	$12G$	$24B$	$12H$	$12I$	$14C$	$28A$
$\|C_{\bar{G}}(g)\|$	168	56	256	128	32	256	80	40	80	22	22	48	48	48	48	24	24	28	28
χ_{42}	0	0	0	0	0	0	0	0	0	1	-1	-1	1	1	-1	0	0	0	0
χ_{43}	0	0	-2	2	0	0	2	-2	0	0	0	-1	1	1	-1	0	0	0	0
χ_{44}	0	0	0	0	-2	2	-2	2	0	0	0	1	-1	-1	1	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0
χ_{46}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	1	1	-1	-1	1	1	-1	0	0	2	0	0	-2	1	-1	0	0
χ_{49}	0	0	-1	-1	1	1	1	1	-1	0	0	0	2	-2	0	-1	1	0	0
χ_{50}	0	0	1	1	-1	-1	0	0	0	0	0	-1	-1	1	1	-1	1	0	0
χ_{51}	0	0	-1	-1	1	1	0	0	0	0	0	-1	-1	1	1	1	-1	0	0
χ_{52}	0	0	-1	-1	1	1	0	0	0	0	0	0	-2	2	0	0	0	0	0
χ_{53}	0	0	1	1	-1	-1	0	0	0	0	0	-2	0	0	2	0	0	0	0
χ_{54}	0	0	1	1	-1	-1	-1	-1	1	0	0	1	1	-1	-1	0	0	0	0
χ_{55}	0	0	-1	-1	1	1	-1	-1	1	0	0	1	1	-1	-1	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	1	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	1	-1	0	0
χ_{58}	0	0	0	0	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	1	1	-1	0	0	-1	-1	1	1	0	0	0	0
χ_{60}	0	0	0	0	0	0	1	1	-1	0	0	-1	-1	1	1	0	0	0	0
χ_{61}	0	0	0	0	0	0	2	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	-2	-2	2	0	0	-1	-1	1	1	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0
χ_{64}	0	0	0	0	0	0	-1	-1	1	0	0	-1	1	-1	1	0	0	0	0
χ_{65}	0	0	0	0	0	0	-1	-1	1	0	0	1	-1	1	-1	0	0	0	0
χ_{66}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	1	-1	0	0
χ_{67}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	-1	1	0	0
χ_{68}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	-1	1	0	0
χ_{69}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	1	-1	0	0
χ_{70}	0	0	0	0	0	0	1	1	-1	0	0	-1	1	-1	1	0	0	0	0
χ_{71}	0	0	0	0	0	0	1	1	-1	0	0	1	-1	1	-1	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0
χ_{76}	0	0	0	0	0	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$14D$	$28B$	$15A$	$30A$	$15B$	$30B$	$21A$	$21B$	$23A$	$23B$
$\lvert C_{\bar{G}}(g)\rvert$	28	28	30	30	30	30	21	21	23	23
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	0	0	0	0	0	0	-1	-1	0	0
χ_3	$-/A$	$-/A$	0	0	0	0	A	$/A$	-1	-1
χ_4	$-A$	$-A$	0	0	0	0	$/A$	A	-1	-1
χ_5	0	0	D	D	$/D$	$/D$	0	0	1	1
χ_6	0	0	$/D$	$/D$	D	D	0	0	1	1
χ_7	0	0	-1	-1	-1	-1	0	0	-1	-1
χ_8	-1	-1	0	0	0	0	1	1	0	0
χ_9	0	0	1	1	1	1	0	0	0	0
χ_{10}	0	0	0	0	$0-$	0	0	0	E	$/E$
χ_{11}	0	0	0	0	0	0	0	0	$/E$	E
χ_{12}	$/A$	$/A$	0	0	0	0	A	$/A$	1	1
χ_{13}	A	A	0	0	0	0	$/A$	A	1	1
χ_{14}	-1	-1	0	0	0	0	-1	-1	0	0
χ_{15}	0	0	0	0	0	0	$-A$	$-/A$	0	0
χ_{16}	0	0	0	0	0	0	$-/A$	$-A$	0	0
χ_{17}	0	0	0	0	0	0	1	1	0	0
χ_{18}	0	0	1	1	1	1	0	0	0	0
χ_{19}	1	1	-1	-1	-1	-1	1	1	0	0
χ_{20}	0	0	0	0	0	0	-1	-1	0	0
χ_{21}	-1	-1	0	0	0	0	1	1	0	0
χ_{22}	1	1	0	0	0	0	-1	-1	1	1
χ_{23}	0	0	0	0	0	0	0	0	0	0
χ_{24}	0	0	-1	-1	-1	-1	0	0	1	1
χ_{25}	0	0	1	1	1	1	0	0	0	0
χ_{26}	0	0	0	0	0	0	0	0	-1	-1
χ_{27}	1	-1	0	0	0	0	0	0	0	0
χ_{28}	-1	1	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0
χ_{31}	$/A$	$-/A$	0	0	0	0	0	0	0	0
χ_{32}	$-/A$	$/A$	0	0	0	0	0	0	0	0
χ_{33}	A	$-A$	0	0	0	0	0	0	0	0
χ_{34}	$-A$	A	0	0	0	0	0	0	0	0
χ_{35}	-1	1	0	0	0	0	0	0	0	0
χ_{36}	1	-1	0	0	0	0	0	0	0	0
χ_{37}	1	-1	0	0	0	0	0	0	0	0
χ_{38}	-1	1	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0

Table 8: *(Continued)*

$[g]_{\bar{G}}$	$14D$	$28B$	$15A$	$30A$	$15B$	$30B$	$21A$	$21B$	$23A$	$23B$
$\lvert C_{\bar{G}}(g)\rvert$	28	28	30	30	30	30	21	21	23	23
χ_{43}	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	1	-1	1	-1	0	0	0	0
χ_{49}	0	0	1	-1	1	-1	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	-1	1	-1	1	0	0	0	0
χ_{55}	0	0	-1	1	-1	1	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	1	-1	1	-1	0	0	0	0
χ_{59}	0	0	D	$-D$	$/D$	$-/D$	0	0	0	0
χ_{60}	0	0	$/D$	$-/D$	D	$-D$	0	0	0	0
χ_{61}	0	0	-1	1	-1	1	0	0	0	0
χ_{62}	0	0	1	-1	1	-1	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0

In Table 8, we take $A = \dfrac{-1+\sqrt{-7}}{2}, B = -1+\sqrt{-7}, C = \dfrac{-3+3\sqrt{-7}}{2}$, $D = \dfrac{-1+\sqrt{-15}}{2}$ and $E = \dfrac{-1+\sqrt{-23}}{2}$ where $/A$, $/B$, $/C$ $/D$ and $/E$ are conjugates of A, B, C D and E respectively.

Table 9: Fusion of $2^{11}{:}M_{24}$ into Co_1

$[x]_{\overline{G}}$	$\rightarrow$	$[y]_{CO_1}$	$[x]_{\overline{G}}$	$\rightarrow$	$[y]_{CO_1}$	$[x]_{\overline{G}}$	$\rightarrow$	$[y]_{CO_1}$	$[x]_{\overline{G}}$	$\rightarrow$	$[y]_{CO_1}$
$1A$		$1A$	$4L$		$4D$	$8F$		$8D$	$14B$		$14B$
$2A$		$2A$	$4M$		$4F$	$8G$		$8D$	$14C$		$14B$
$2B$		$2c$	$4N$		$4E$	$8H$		$8F$	$14D$		$14B$
$2C$		$2A$	$4O$		$4F$	$8I$		$8E$	$15A$		$15D$
$2D$		$2C$	$5A$		$5B$	$8J$		$8E$	$15B$		$15D$
$2E$		$2C$	$6A$		$6E$	$8K$		$8F$	$16B$		$16A$
$2F$		$2B$	$6B$		$6C$	$10A$		$10D$	$16B$		$16B$
$3A$		$3B$	$6C$		$6G$	$10B$		$10F$	$11A$		$11A$
$3B$		$3D$	$6D$		$6I$	$10C$		$10C$	$20A$		$20B$
$4A$		$4A$	$6E$		$6E$	$10D$		$10F$	$21A$		$21C$
$4B$		$4B$	$6F$		$6G$	$12A$		$12E$	$21B$		$21C$
$4C$		$4C$	$6G$		$6H$	$12B$		$12G$	$22A$		$22A$
$4D$		$4D$	$6H$		$6I$	$12C$		$12G$	$23A$		$23A$
$4E$		$4B$	$7A$		$7B$	$12D$		$12G$	$23B$		$23B$
$4F$		$4D$	$7B$		$7B$	$12E$		$12I$	$24A$		$24C$
$4G$		$4F$	$8A$		$8A$	$12F$		$12J$	$24B$		$24E$
$4H$		$4B$	$8B$		$8B$	$12G$		$12M$	$28A$		$28A$
$4I$		$4D$	$8C$		$8B$	$12H$		$12L$	$28B$		$28A$
$4J$		$4F$	$8D$		$8C$	$12I$		$12M$	$30A$		$30D$
$4K$		$4C$	$8E$		$8D$	$14A$		$14B$	$30B$		$30D$

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