AUT Journal of Mathematics and Computing

AUT J. Math. Comput., 5(1) (2024) 71-80
https://doi.org/10.22060/AJMC.2023.21635.1109

Original Article

A combined Bernoulli collocation method and imperialist competitive algorithm for optimal control of sediment in the dam reservoirs

Raheleh Khanduzi*[a], Asyieh Ebrahimzadeh[b], Zahra Ebrahimzadeh[c]

[a] *Department of Mathematics and Statistics, Gonbad Kavous University, P.O. Box 49717-99151, Gonbad Kavous, Golestan, Iran*
[b] *Department of Mathematics Education, Farhangian University, P.O. Box 14665-889, Tehran, Iran*
[c] *PhD student, Department of Natural Engineering, Faculty of Natural Resources and Earth Sciences, Shahrekord University, Shahrekord, Iran*

ABSTRACT: Reservoir sedimentation increases economic cost and overflow of dam water. An optimal control problem (OCP) with singularly perturbed equations of motion is perused in the fields of sediment management during a finite lifespan. Subsequently the OCP is transformed to a nonlinear programming problem by utilizing a collocation approach, and then we employed the imperialist competitive algorithm to improve the execution time and decision. So, the solutions of the optimal control and fast state as well as the maximization of net present value of dam operations are obtained. An illustrative practical study demonstrated that sedimentation management is economically favourable for volume of confined water and total amount in remaining storage and effectiveness of the propounded approach.

Review History:

Received:19 September 2022
Revised:15 January 2023
Accepted:20 March 2023
Available Online:01 January 2024

Keywords:

Optimal control
Singularly perturbed differential equation
Reservoir sedimentation
Collocation method
Imperialist competitive algorithm

AMS Subject Classification (2010):

9J21; 65N35; 97R40

1. Introduction

The sedimentation in reservoirs is an inevitable phenomenon that arises from the soil erosion above the dam. The erosion occurs when the current velocity and tensile stress on the river bed are greater than the specified threshold. The removal of sediments in the reservoir has undesirable effects such as reducing the reservoir capacity, resulting in hydropower generation, navigation, water supply, menacing resistance of dam and increasing the probability of overtopping happening. Different factors influence the removal of sediment such as storage capacity, proportional evaporation, operating and conservation costs, average sediment inflow per year, water required to delete a unit of sediment, reservoir height, slope of sediment transport-height. Motivated by these topics, we develop and propose a

**Corresponding author.*
E-mail addresses: khanduzi@gonbad.ac.ir, a.ebrahimzadeh@cfu.ac.ir, z.brahimzadeh@stu.sku.ac.ir

new hybrid method depend on collocation method and imperialist competitive algorithm (ICA) to solve the OCP of reservoir sedimentation on an application example. Before further clarifying the mathematical model and solution approach, we first outline an overview of related studies on the OCP of reservoir sedimentation and highlight the main objectives. Following up on this issue, most researches have been conducted recently on the optimal control problem (OCP) of reservoir sedimentation. Carriaga and Mays [4] was proposed a discrete-time OCP based on a differential dynamic programming to specify the release management of an upstream reservoir. The aim of the problem was to minimize the bed erosion and degradation on the downstream river. Zhu et al. [22] introduced an OCP of sedimentation management to find optimal sites and timing of dredging for minimizing total cost and getting a channel in which water depths were not less than defined values. Then, the OCP was converted into an optimization problem utilizing a penalty function algorithm to delete explicit constraints on the state variables. The reduced problem was solved by the conjugate gradient technique. Palmieri et al. [18] developed an OCP for evaluating the economic practicality of sediment management approaches that would permit the lifespan of dams to be continued endlessly.

Nicklow and Muleta [17] modelled an optimal control methodology based on a genetic algorithm (GA) to minimize sediment yield arising from watersheds. Nicklow and Bringer [16] developed a discrete-time OCP and a GA to control surplus sedimentation in multi-reservoir river networks. Valizadegan et al. [21] proposed an OCP to assess the minimization of reservoir sedimentation. The sedimentation problem in the reservoirs was simulated using GSTARS software. The resulting optimization problem was solved by the direct search algorithm. A simulation model was executed to solve the governing hydraulic and sediment restrictions, while the GA was utilized to solve the overall optimization problem. Ding and Wang [7] introduced a simulation-based mathematical programming to methodically consider optimal control of flood levels for different geometries and sediment characteristics in alluvial channels. Ding et al. [6] formulated a bi-objective optimization model to obtain optimal control of flows and sediment transport in a sedimentary river or a watershed to minimize flood water levels and morphological variations under operational constraints. Alvarez-Vazquez et al. [1] dealt with an OCP of partial differential equations to control the sedimentation of suspended particles in large streams. The main objective of this model was to provide the optimal management of a waterway to prevent the settlement of suspended particles and pathway failure, undesired growth of plant life.

As mentioned above, the optimal control techniques are applied to evaluate sedimentation management strategies. The various discretization schemes are used to convert OCP to a mathematical programming problem and generate approximate solutions converge to the exact solution. Now observe that in order for obtaining the solutions with high decision and short execution time, the appropriate optimization algorithms should be applied. As reported by Huffaker and Hotchkiss [10], the reduced optimization problem is nonlinear and the numerical algorithms may be trapped on the local maximum and do not converge to a global solution. This fact arises due to the presence of a high-dimensional and nonlinear objective function. So, it is necessary to detect a reliable algorithm that can escape from the local maximum and converge to the global maximum. Hence, Atashpaz-Gargari and Lucas [3] introduced ICA on the basis of imperialistic competition. Among all the novel natural-inspired optimization methods, the ICA is chosen for the optimization problem that dealt with finding control variable of impounded water for reservoir sedimentation management because of acceptable performance on benchmark test functions. This method was also utilized very well on the various optimization problems (Kaveh and Talatahari, [11]; Nazari-Shirkouhi et al, [15]; Shabani et al, [19]; Dossary and Nasrabadi, [8]; Khalilnejad, [12]; Lei, [5]). Based on the above mentioned advantages of this method, it is expected to provide satisfactory solutions for the OCP of reservoir sedimentation.

In the present study, we introduce a collocation method for converting OCP to a nonlinear programming problem (NLP). Some advantages of the collocation method are as follows:

1. It can be utilized for a larger range of OCPs, especially those with more complex system and geometry.
2. In this problem, we don't have an exact solution, but the spectral method in the OCP is convergent according to [9, 14].The solutions are trustworthy and match with reality.
3. The solution is found directly at the collocation points.
4. The implementation of Derivative Bernoulli operational matrices method is very simple. One of the predominant advantages of using derivative operational matrix is that the matrix D, introduced in (11), has large numbers of zero elements, and they are sparse; hence, the present method is very efficient. The other advantageous of utilizing this matrix is that in equation (11) for the derivative operational matrix, the relation is established equally, not approximately.
5. The Bernoulli formula can then be applied to find the value of control variables at any point. To the best of our knowledge, this is the first paper to adapt this approach based on collocation method and ICA in solving the OCP of reservoir sedimentation so far.

This study is outlined as follows: In Section 2 we describe a detailed mathematical model of OCP. In Section 3 we propose a solution methodology to solve this problem. In Section 4 we provide the application example of OCP of reservoir sedimentation and numerical simulations. Section 6 summarizes our conclusions.

2. Model Description

The following entities are used in the mathematical model for the optimal control problem of sediment reduction in reservoirs.

2.1. Notations

The notations used in this mathematical model are described below:

2.1.1. Parameters

r	Discount factor
s_0	Initial storage volume
L	Evaporation factor
α	Composite discount rate
om	Operating and maintenance costs
η	Average annual sediment flux
γ	Ratio of water required to eliminate a unit of sediment
p_h	Unit net profit from eliminating sediment
p_c	Unit net profit from water consumption
e^m	Maximum reservoir height
$F_1(F_2)$	Slope (intercept) of sediment transport-height
ϕ_1	Composite constant
ϕ_2	Composite constant
k_1	Height-storage water performance score
$c^{max}(c^{min})$	Maximum (minimum) consumption factor
ε	Perturbation constant
V	Cost incurred by the loss of the dam

2.1.2. Variables

s_t	Remaining storage volume at time t (state variable)
w_t	The amount of impounded water at time t (state variable)
c_t	Consumption amount of storage water in non-hydrosuction process at time t (control variable)

2.2. Functions

$e(w,s)$	Height-storage
$x(e)$	Sediment transport-height
$R(w,s)$	Reservoir refill operating
$y(x)$	Water extracted from hydrosuction pipeline
$ev(w)$	Evaporative water loss
$D(s)$	Sediment settling

These functions are defined as follows:

Concerning with the performance of observational height-storage curves, is modelled as a Michaelis–Menton function:

$$e(w_t, s_t) = \frac{e^{max} w_t}{w_t + D(s_t)}, D(s_t) = k_1 s_t \tag{1}$$

The transport-height function, i.e., $x(e_t)$ is calculated as

$$x(e_t) = F_1 e_t + F_2 \tag{2}$$

Replacing (1) into (2) modifies the sediment transport factor as a mathematical relationship of the two state variables (w_t and s_t):

$$x(w_t, s_t) = \frac{\phi_1 w_t}{w_t + D(s_t)} + F_2, \phi_1 = e^{max} F_1$$

Water extracted from hydrosuction pipeline, i.e.,$y(x_t)$, is directly proportional to the sediment transport factor:

$$y(x_t) = \gamma x(w_t, s_t)$$

The reservoir has a storage region for annual extra water, $R(w_t, s_t)$, corresponding to the difference between resting storage volume and the amount of water impounded :

$$R(w_t, s_t) = s_t - w_t$$

Evaporative losses for each year, $ev(w_t)$, are estimated as a fixed fraction of impounded water:

$$ev(w_t) = Lw_t$$

Also, the time-dependent changes of confined water and storage volume are defined by

$$\frac{dw}{dt} = R(w_t, s_t) - c_t - y(x_t) - ev(w_t) \tag{3}$$

$$\frac{ds}{dt} = \varepsilon(x(w_t, s_t) - \eta) \tag{4}$$

In relation (3), the amount of confined water varies at a net factor during a year correspond to the difference between refill for each year and consumptive activities. These involve the factors of impounded-water utilization in non-hydrosuction activities, impounded-water utilization in hydrosuction dredging, and evaporative losses. In relation (4), the annual net factor of change in storage volume is the difference between the sediment transport factor and a constant factor for each year at which sediment is involved in the reservoir. Relations (3) and (4) comprise a structure of 'singularly perturbed' differential equations. The flow of net revenues for each year from the dam/reservoir scheme is:

$$p_c c_t + p_h x(w_t, s_t) - om$$

The $p_c c_t$ shows net revenue of each year from the utilization of impounded water in non-hydrosuction activities and $p_h x(w_t, s_t)$ indicates annual net revenue from sediment transport.

2.3. Mathematical model

The mathematical model of the optimal control problem for reservoir sedimentation management is represented as follows.

$$\max_{c_t} \int_0^T e^{-rt}(p_c c_t + p_h x(w_t, s_t) - om)dt + e^{-rt}V \tag{5}$$

$$\frac{dw}{dt} = R(w_t, s_t) - c_t - y(x_t) - ev(w_t), w(t=0) = w_0, \frac{dw}{dt} = R(w_t, s_t) - c_t - y(x_t) - ev(w_t), s(t=0) = s_0$$

$$c^{min} \leq c_t \leq c^{max}$$

where w_0 and s_0 are primary requirements on the state variables. The T is the dam lifespan, and $e^{-rt}V$ is a discounted salvage value relating to the designer's decision at the dam's loss. Problem (5) is a rapid approach problem as considered in Spence and Starrett [20]. It needs upper and lower limits on the control variable: $c^{min} \leq c_t \leq c^{max}$.

3. Solution Methodology

3.1. Collocation method

Bernoulli polynomials of order m can be described with the following equation (Costabile, Dellaccio and Gualtieri, [13])

$$\Psi_m(t) = \sum_{i=0}^{m} \binom{m}{i} \alpha_{m-i} t^i$$

in which $\alpha_i (i = 0, ..., m)$ are Bernoulli numbers. These number are satisfied in the following expansion (Arfken and Weber, [2])

$$\sum_{k=0}^{n} \binom{k}{n+1} \alpha_i(t) = (n+1)t^n$$

The Bernoulli vector is defined in the form

$$\beta^T(t) = [\Psi_0(t), \Psi_1(t), ..., \Psi_{m-1}(t)]$$

The derivative operational matrix of $\beta(t)$ with the aid of wellknown relations

$$\frac{d\Psi_i(t)}{dt} = i\Psi_{i-1}(t) \quad , \quad i \geq 1$$

$$\int_0^1 \Psi_i(t)dt = 0 \quad , \quad i \geq 1$$

can be defined in the matrix form by

$$\beta^{'}(t) = D\beta(t) \tag{6}$$

where D is

$$\begin{pmatrix} 0 & 0 & 0 & . & . & . & 0 & 0 & 0 \\ 1 & 0 & 0 & . & . & . & 0 & 0 & 0 \\ 0 & 2 & 0 & . & . & . & 0 & 0 & 0 \\ . & . & . & . & & & . & . & . \\ . & . & . & & & . & . & . & . \\ . & . & . & & & . & . & . & . \\ 0 & 0 & 0 & . & . & . & N-1 & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & N & 0 \end{pmatrix}_{(m+1)(m+1)}.$$

We utilize the collocation method based on Bernoulli matrix of derivative to solve the OCP. Firstly, the solution of (5) is approximated by the Bernoulli polynomials.

$$s_t = s^T\beta(t),$$

$$w_t = w^T\beta(t),$$

$$c_t = c^T\beta(t),$$

The vectors s^T, w^T and c^T is defined as follows

$$s^T = [s_0, s_1, ..., s_{m-1}], \tag{7}$$

$$w^T = [w_0, w_1, ..., w_{m-1}],$$

$$c^T = [c_0, c_1, ..., c_{m-1}],$$

Making use of (6) yield

$$\frac{dw}{dt} = w^T D\beta(t), \tag{8}$$

$$\frac{ds}{dt} = s^T D\beta(t), \tag{9}$$

By substituting the equation (7), (8) and (9) in ordinary differential equations (5), we gain

$$w^T D\beta(t) = R(w^T\beta(t), s^T\beta(t)) - c^T\beta(t) - \gamma x(w^T\beta(t), s^T\beta(t)) - Lw^T\beta(t)$$

$$s^T D\beta(t) = \varepsilon[x(w^T\beta(t), s^T\beta(t)) - \eta]$$

$$c^{min} \leq c^T\beta(t) \leq c^{max}$$

$$w^T\beta(0) = w_0$$

$$s^T\beta(0) = s_0$$

Now, we approximate the objective function (5)

$$\int_0^T e^{-rt}(p_c c^T\beta(t) + p_h x(w^T\beta(t), s^T\beta(t)) - om)dt + e^{-rt}V \tag{10}$$

Now, the integration (10) is approximated by numerical integration

$$\sum_{i=0}^{q} w_i e^{-rt}(p_c c^T\beta(t_i) + p_h x(w^T\beta(t_i), s^T\beta(t_i)) - om)dt + e^{-rt}V$$

where w_i and t_i are weights and nodes of integration. Now, the resulted NLP is

$$\max \sum_{i=0}^{q} w_i e^{-rt}(p_c c^T \beta(t_i) + p_h x(w^T \beta(t_i), s^T \beta(t_i)) - om)dt + e^{-rt}V \tag{11}$$

$$w^T D\beta(t) = R(w^T \beta(t), s^T \beta(t)) - c^T \beta(t) - \gamma x(w^T \beta(t), s^T \beta(t)) - Lw^T \beta(t)$$
$$s^T D\beta(t) = \varepsilon[x(w^T \beta(t), s^T \beta(t)) - \eta]$$
$$c^{min} \leq c^T \beta(t) \leq c^{max}$$
$$w^T \beta(0) = w_0$$
$$s^T \beta(0) = s_0$$

3.2. Imperialist competitive algorithm

Imperialist competitive algorithm (ICA), first introduced by Atashpaz-Gargari and Lucas [3] for solving different types of optimization problems. This algorithm is an evolutionary algorithm that simulates the competition between imperialists to possess more colonies in order to increase their empires subject to series of actions to compete imperialists. ICA begins with an initial population of countries. The percentage of the best countries (usually 10%) in the population choose to be the imperialists and the reminder of population comprise the colonies of these imperialists. In ICA, the more strong imperialist, have the more colonies. When the competition begins, imperialists try to attain more colonies and the colonies are starting to advance toward their imperialists. Therefore, the strong imperialists will be developed in the course of the competition and the weak ones will be failed. Then, one imperialist will survive in the final step. In this step, the situation of imperialist and its colonies will be the identical.

The flowchart of the ICA is graphed in Figure 1. More details about the ICA are introduced in (Atashpaz-Gargari and Lucas, [3]; Kaveh and Talatahari, [11]; Nazari-Shirkouhi et al, [15]; Shabani et al, [19]; Dossary and Nasrabadi, [8]; Khalilnejad, [12]; Lei, [5]). As previously stated in this mathematical model, the objective function of ICA is the negative value of (5). Decision variables are the remaining storage volume (s_t), amount of confined water (w_t), and consumption factor of storage water in non-hydrosuction activities (c_t). The number of countries is 100 and the number of imperialists is 10.

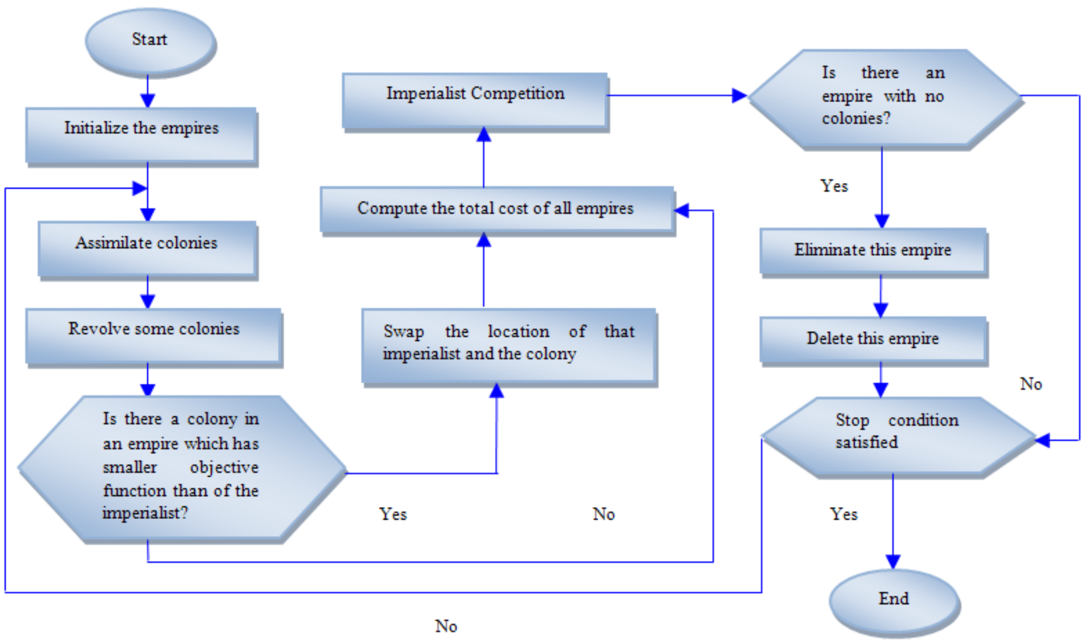

Figure 1: Flowchart of the imperialist competitive algorithm

4. Application Example

This section includes the numerical results obtained with collocation method and ICA. The collocation approach are coded in MATLAB (R2018b) for converting OCP to a nonlinear programming problem (NLP) and then NLP is solved using ICA in MATLAB on a 64-bit computer, Intel Core i7, 3.3 GHz processor and 4 GB of RAM.

In order to obtain the remaining storage volume, amount of confined water and consumption rate of storage water in non-hydrosuction activities, an application example is studied. Operations and maintenance costs computed by: (1) multiplying the calculated cost for a given unit of dam building (1.39$/m3) by to get total cost of dam building; and (2) considering that operations and maintenance cost per each year is 2% of total cost. Slope (intercept) of sediment transport-height functions are approximated utilizing data linking sediment transport by way of a hydrosuction pipeline to changes in hydraulic height. The total length of pipelines, pipeline diameter, and a range of equipment to construct storage reservoirs are considered. The problem parameters are given in Table 1. We

Table 1: Hydraulic and economic parameter values

Parameters	Units	Values
r	1/year	0.05
s_0	m^3	60×10^6
L	1/year	0.02
α	1/year	$r + L + 1$
om	\$/year	5004
η	$m^3/year$	880000
γ	-	16
p_h	$\$/m^3$	20
p_c	$\$/m^3$	0.97
e_m	m	53
$F_1(F_2)$	$m^3/year/m$ $(m^3/year)$	84982.5 (-184318)
ϕ_1	-	$e^m F_1$
ϕ_2	-	$\phi_1/\alpha p_c$
k_1	-	0.08
$c^{max}(c^{min})$	$m^3/year$	$60 \times 106(0)$
ε	-	0.07

execute a collocation approach with ICA to evaluate the solution to optimality mathematical model of (5) and (11). The remaining storage volume function, measuring the amount of impounded water and consumption factor of storage water in non-hydrosuction activities are plotted based on time in Figs. 2-4. It compares well to the graphical depiction in the variables, which also decreases with amount of confined water at an annual growth rate [Fig. 2] and grows with storage volume at a water reduction rate [Fig. 3]. Fig. 4 demonstrates the dynamic of storage water variable on non-hydrosuction activities. The diagram of is negative when storage capacity is lower than the low-capacity. Positive consumption rates are due to the best large-capacity.

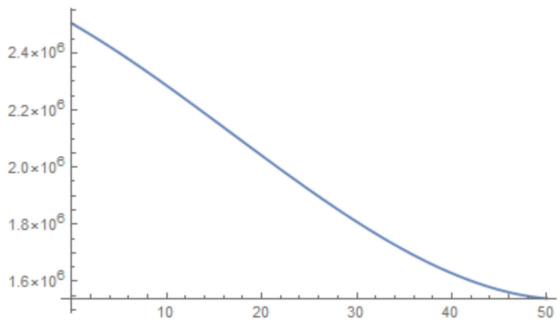

Figure 2: Graphical depiction for volume of impounded water based on time (w_t)

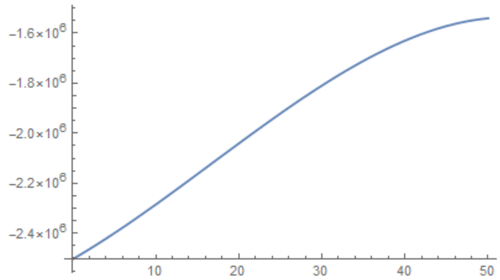

Figure 3: Graphical depiction for remaining storage capacity based on time (s_t)

Figs. 5-7 demonstrate dynamics of sediment perching, evaporative losses and reservoir refill. Graphically, Fig. 5 show that the process of sediment perching increases in the course of fifty years as a bad outcome of reservoir sedimentation. So, it highlights the good performance of hydrosuction dredging at this time and has an important

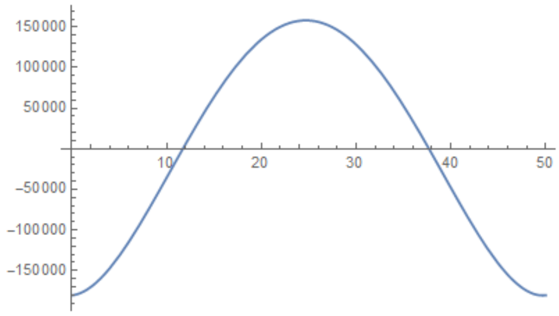

Figure 4: Graphical depiction for consumption factor of storage water in non-hydrosuction activities based on time (c_t)

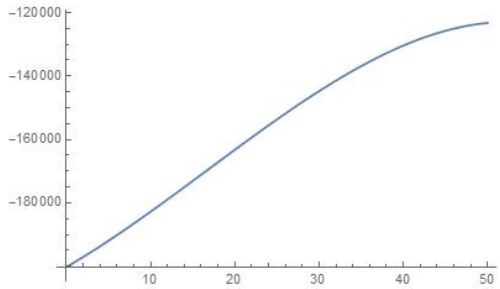

Figure 5: Dynamic of sediment perching for variable s_t ($D(s_t)$)

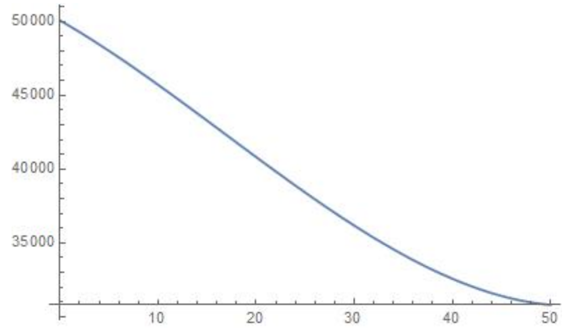

Figure 6: Dynamic of evaporative losses for variable w_t ($ev(w_t)$)

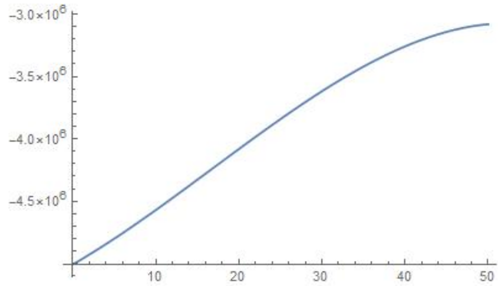

Figure 7: Dynamic of reservoir refill for variables w_t, s_t ($R(w_t, s_t)$)

influence in determining the financial dynamics of hydrosuction dredging. A gradual increase produces a positive insignificant effect by admitting for developed usage, but obtains an offsetting negligible influence by reducing the performance of sediment transport with respect to decreased sediment perching. Fig. 6 indicates annual evaporative losses as a relative amount of a reservoir's water level. It shows a vital decrease in evaporative losses over the 50 years. According to this figure, evaporative losses from reservoirs decrease from 50000 (m3) to 0 (m3) for a considerable period. This means that the increase of storage capacity has the impact on yield reduction in evaporation losses. Fig. 7 shows that reservoir refill increases over 50 years. The reservoir has storage area for extra water per annum, corresponding to the difference between surplus storage volume and the amount of water earlier dehydrated. Annual water inflow further on than storage capacity is dropped. Therefore, there is a one-to-one tradeoff between residual water released into the reservoir and refill chance lost to uncontrolled release. This forms a refill chance cost of dehydrating concerning an additive unit of storage water. It obtains a one-to-one loss in the capability of water inflow per year to refill the reservoir.

5. Conclusion

This paper developed a hybrid method concerning a collocation method and imperialist competitive algorithm (ICA) for solving an optimal control problem (OCP), in which optimal decisions have been made for the reservoir sedimentation management. The problem was formulated as an OCP with singularly perturbed equations of motion. The major contribution of this study was the simultaneous consideration to an OCP of reservoir sedimentation with respect to storage capacity, evaporation, operating and maintenance costs, sediment inflow and water required to eliminate a unit of sediment as well as a proposed approach for the solution of application example with high accuracy.

Acknowledgements

The first author has been supported by the financial support of the research-technology management of Gonbad Kavous University (Grant no. 6/483).

References

[1] L. Alvarez-Vázquez, A. Martínez, C. Rodríguez, and M. Vázquez-Méndez, *Sediment minimization in canals: An optimal control approach*, Math. Comput. Simul., 149 (2018), pp. 109–122.

[2] G. B. Arfken, *Mathematical Methods for Physicists*, Academic Press, third ed., 1985.

[3] E. Atashpaz-Gargari and C. Lucas, *Imperialist competitive algorithm: An algorithm for optimization inspired by imperialistic competition*, in 2007 IEEE Congress on Evolutionary Computation, 2007, pp. 4661–4667.

[4] C. C. Carriaga and L. W. Mays, *Optimal control approach for sedimentation control in alluvial rivers*, J. Water Resour. Plan. Manag, 121 (1995), pp. 408–417.

[5] J. C. Deming Lei, Yue Yuan and D. Bai, *An imperialist competitive algorithm with memory for distributed unrelated parallel machines scheduling*, Int. J. Prod. Res., 58 (2020), pp. 597–614.

[6] Y. Ding, M. Elgohry, M. S. Altinakar, and S. S. Wang, *Optimal control of flow and sediment in river and watershed*, in Proceedings of 2013 IAHR Congress, Tsinghua University Press, Chengdu, China, vol. 813, 2013.

[7] Y. Ding and S. S. Wang, *Optimal control of flood water with sediment transport in alluvial channel*, Separation and Purification Technology, 84 (2012), pp. 85–94.

[8] M. A. A. Dossary and H. Nasrabadi, *Well placement optimization using imperialist competitive algorithm*, J. Pet. Sci. Eng., 147 (2016), pp. 237–248.

[9] A. Ebrahimzadeh and R. Khanduzi, *A directed tabu search method for solving controlled Volterra integral equations*, Math. Sci. (Springer), 10 (2016), pp. 115–122.

[10] R. Huffaker and R. Hotchkiss, *Economic dynamics of reservoir sedimentation management: Optimal control with singularly perturbed equations of motion*, J. Econ. Dyn. Control., 30 (2006), pp. 2553–2575.

[11] A. Kaveh and S. Talatahari, *Optimum design of skeletal structures using imperialist competitive algorithm*, Computers & Structures, 88 (2010), pp. 1220–1229.

[12] A. Khalilnejad, A. Sundararajan, and A. I. Sarwat, *Optimal design of hybrid wind/photovoltaic electrolyzer for maximum hydrogen production using imperialist competitive algorithm*, J. Mod. Power Syst. Clean Energy, 6 (2018), pp. 40–49.

[13] D. H. Lehmer, *A new approach to Bernoulli polynomials*, Amer. Math. Monthly, 95 (1988), pp. 905–911.

[14] K. Maleknejad and A. Ebrahimzadeh, *An efficient hybrid pseudo-spectral method for solving optimal control of Volterra integral systems*, Math. Commun., 19 (2014), pp. 417–435.

[15] S. Nazari-Shirkouhi, H. Eivazy, R. Ghodsi, K. Rezaie, and E. Atashpaz-Gargari, *Solving the integrated product mix-outsourcing problem using the imperialist competitive algorithm*, Expert Syst. Appl., 37 (2010), pp. 7615–7626.

[16] J. W. Nicklow and J. A. Bringer, *Optimal Control of Sedimentation in Multi-Reservoir River Systems Using Genetic Algorithms*, pp. 1–10.

[17] J. W. Nicklow and M. K. Muleta, *Watershed management technique to control sediment yield in agriculturally dominated areas*, Water International, 26 (2001), pp. 435–443.

[18] A. Palmieri, F. Shah, and A. Dinar, *Economics of reservoir sedimentation and sustainable management of dams*, Journal of environmental management, 61 (2001), pp. 149–163.

[19] H. Shabani, B. Vahidi, and M. Ebrahimpour, *A robust pid controller based on imperialist competitive algorithm for load-frequency control of power systems*, ISA Transactions, 52 (2013), pp. 88–95.

[20] A. M. Spence and D. Starrett, *Most Rapid Approach Paths in Accumulation Problems*, International Economic Review, 16 (1975), pp. 388–403.

[21] E. Valizadegan, M. Bajestan, H. Mohammad, and H. Mohammad Vali Samani, *Control of sedimentation in reservoirs by optimal operation of reservoir releases*, Journal of Food, Agriculture and Environment, 7 (2009), pp. 759–763.

[22] J. Zhu, Q. Zeng, D. Guo, and Z. Liu, *Optimal control of sedimentation in navigation channels*, J. Hydraulic Eng., 125 (1999), pp. 750–759.

Please cite this article using:

Raheleh Khanduzi, Asyieh Ebrahimzadeh, Zahra Ebrahimzadeh, A combined Bernoulli collocation method and imperialist competitive algorithm for optimal control of sediment in the dam reservoirs, AUT J. Math. Comput., 5(1) (2024) 71-80
https://doi.org/10.22060/AJMC.2023.21635.1109