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Original Article

Generalized η-Ricci solitons on f-Kenmotsu 3-manifolds associated to the Schouten-van Kampen connection

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ABSTRACT: In this paper, we investigate f-Kenmotsu 3-dimensional manifolds admitting generalized η-Ricci solitons with respect to the Schouten-van Kampen connection. We provide an example of generalized η-Ricci solitons with respect to the Schouten-van Kampen connection on an f-Kenmotsu 3-dimensional manifold to prove our results.

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1. Introduction

The Kenmotsu manifold was introduced by Kenmotsu [20] in 1972 as new class of almost contact metric manifolds. Then, Olszak and Rosca [27] introduced f-Kenmotsu manifolds. By an f-Kenmotsu manifold we mean an almost contact metric manifold which is normal and locally conformal almost cosymplectic. The Schouten-van Kampen connection have been introduced for a study of non-holomorphic manifolds [31, 38]. Recently, Bjenancu [2] investigates Schouten-van Kampen connection on foliated manifolds. Olszak [26] study Schouten-van Kampen connection on almost contact metric structure. Many authors studied some calsses of almost contact metric manifolds with respecto to the Schouten-van Kampen connection [16, 19, 21, 28, 42].

On the other hand, the notion of Ricci flow on a Riemannian manifold introduced by Hamilton [17] and it is defined by

$$\frac{\partial}{\partial t}g = -2S$$

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where S is the Ricci tensor of a manifold. The special solutions of the Ricci flow equation are called Ricci solitons which are generalization of Einstein metrics. A Ricci soliton [15] is a triplet (g, V, λ) on a pseudo-Riemannian manifold M such that

$$\mathcal{L}_V g + 2S + 2\lambda g = 0,$$

where $\mathcal{L}_V$ is the Lie derivative along the potential vector field V, S is the Ricci tensor, and λ is a real constant. Ricci solitons are interesting and useful in physics and are often referred as quasi-Einstein [11, 10]. The Ricci soliton is called shrinking, steady and expanding according as λ be negative, zero, positive, respectively. If the vector field V is the gradient of a potential function ψ, then g is called a gradient Ricci soliton. Nurowski and Randall [24] introduced the notion of generalized Ricci soliton as follows

$$\mathcal{L}_V g + 2\mu V^\flat \otimes V^\flat - 2\alpha S - 2\lambda g = 0,$$

where $V^\flat$ is the canonical 1-form associated to V. Also, as a generalization of Ricci soliton, the notion of η-Ricci soliton was introduced by Cho and Kimura [14] which it is a 4-tuple (g, V, λ, ρ), where V is a vector field on M, λ and ρ are constants, and g is a pseudo-Riemannian metric satisfying the equation

$$\mathcal{L}_V g + 2S + 2\lambda g + 2\rho\eta \otimes \eta = 0,$$

where S is the Ricci tensor associated to g. Many authors studied the η-Ricci solitons [5, 4, 6, 18, 22, 29, 36]. In particular, if $\rho = 0$, then the η-Ricci soliton equation reduces to the Ricci soliton equation. Motivated by the above works M. D. Siddiqi [32] introduced the notion of generalized η-Ricci soliton as follows

$$\mathcal{L}_V g + 2\mu V^\flat \otimes V^\flat + 2S + 2\lambda g + 2\rho\eta \otimes \eta = 0.$$

Motivated by [1, 9, 23] and the above studies, we investigate generalized η-Ricci solitons on f-Kenmotsu 3-dimensional manifolds assoicated to the Schouten-van Kampen connection. We give an example of generalized η-Ricci soliton on a f-Kenmotsu 3-dimensional manifold with respect to the Schouten-van Kampen connection.

The paper is orgonaized as follows. In Section 2, we recall some necessary and fundamental concepts and fourmulas on f-Kenmotsu 3-dimensional manifolds which be used throughout the paper. In Section 3, we give the main results and their proofs. In Section 4, we provide an example of an f-Kenmotsu 3-dimensional manifold admits in a generalized η-Ricci soliton with respect to the Schouten-van Kampen connection.

2. Preliminaries

A $(2n + 1)$-dimensional Riemannian manifold (M, g) is called an almost contact metric manifold [7, 8] with an almost contact structure (φ, ξ, η, g), whenever there exist a $(1, 1)$-tensor field φ, a vector field ξ and a 1-form η such that

$$\varphi^2(X) = -X + \eta(X)\xi, \eta(\xi) = 1,$$
$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y),$$

for all vector fields X, Y. In this case, we get $\varphi\xi = 0$, $\eta \circ \varphi = 0$, and $\eta(X) = g(X, \xi)$. The fundamental 2-form Φ of M is given by

$$\Phi(X, Y) = g(X, \varphi Y),$$

for all vector fields X, Y. An almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is said to be an f-Kenmotsu manifold [25] if

$$(\nabla_X \varphi)(Y) = f\{g(\varphi X, Y)\xi - \eta(Y)\varphi Y\}$$

for all vector fields X, Y, where $f \in C^\infty(M)$ such that $df \wedge \eta = 0$. In particular, if $f = c$ is a constant then the manifold becomes an c-Kenmotsu manifold [37]. If $f = 1$ then the manifold is a Kenmotsu manifold [20]. Clearly, an f-Kenmotsu manifold is cosymplectic manifold when $f = 0$. For an f-Kenmotsu manifold we have

$$\nabla_X \xi = f\{X - \eta(X)\xi\}, \tag{1}$$

for any vector field X. Hence,

$$(\nabla_X \eta)Y = f\{g(X, Y) - \eta(X)\eta(Y)\}, \tag{2}$$

for all vector fields X, Y. The condition $df \wedge \eta = 0$ is true if $\dim M \geq 5$. This does not hold in general if $\dim M = 3$ [27].

Using (1), (2), and Weyl tensor in 3-dimensional Riemannian manifolds, we have

$$R(X,Y)\xi = -(f^2+\xi(f))\{\eta(Y)X-\eta(X)Y\},$$
$$R(X,\xi)Y = (f^2+\xi(f))\{g(X,Y)\xi-\eta(Y)X\},$$

for all vector fields X, Y, where R is the Riemannian curvature tensor. The Ricci tensor S of a 3-dimensional f-Kenmotsu manifold M is given by

$$S(X,Y) = (\frac{r}{2}+f^2+\xi(f))g(X,Y) - (3f^2+3\xi(f)+\frac{r}{2})\eta(X)\eta(Y), \tag{3}$$

for all vector fields X, Y, where r is the scalar curvature of M. From (3), we also get

$$S(X,\xi) = -2(f^2+\xi(f))\eta(X), \tag{4}$$

for all vector field X.

Let M be an almost contact metric manifold and TM be the tangent bundle of M. We get two naturally defined distribution on tangent bundle TM as follows

$$H = \ker\eta, \qquad \hat{H} = \mathrm{span}\{\xi\},$$

thus we have $TM = H \oplus \hat{H}$. Hence, by this composition we can define the Schouten-van Kampen connection $\bar{\nabla}$ [3, 33] on M with respect to Levi-Civita connection ∇ as follows

$$\bar{\nabla}_X Y = \nabla_X Y - \eta(Y)\nabla_X\xi + ((\nabla_X\eta)(Y))\xi \tag{5}$$

for all vector fields X, Y. From [28, 33, 34, 35] we get

$$\bar{\nabla}\xi = 0, \qquad \bar{\nabla}\eta = 0,$$

and the torsion $\bar{T}$ of $\bar{\nabla}$ is determined by

$$\bar{T}(X,Y) = \eta(X)\nabla_Y\xi - \eta(Y)\nabla_X\xi + 2d\eta(X,Y)\xi,$$

for all vector fields X, Y. Suppose that $\bar{R}$ and $\bar{S}$ are the curvature tensors and the Ricci tensors of the connection $\bar{\nabla}$, respectively. From [42] on a 3-dimensional f-Kenmotsu manifold we have

$$\bar{\nabla}_X Y = \nabla_X Y + f\big(g(X,Y)\xi - \eta(Y)X\big) \tag{6}$$

and

$$\bar{S}(X,Y) = S(X,Y) + (2f^2+\xi(f))g(X,Y) + \xi(f)\eta(X)\eta(Y), \tag{7}$$

for all vector fields X, Y, where S denotes the Ricci tensor of the connection ∇. Using (7), the Ricci operator $\bar{Q}$ of the connection $\bar{\nabla}$ on a 3-dimensional f-Kenmotsu manifold is given by

$$\bar{Q}X = QX + (2f^2+\xi(f))X + \xi(f)\eta(X)\xi,$$

for all vector field X. Let r and $\bar{r}$ be the scalar curvature of the Levi-Civita connection ∇ and the Schouten-van Kampen connection $\bar{\nabla}$. (7) yields

$$\bar{r} = r + 6f^2 + 4\xi(f).$$

Applying (6) we get

$$\overline{\mathcal{L}}_V g(X,Y) = \mathcal{L}_V g(X,Y) + f\left[g(X,V)\eta(Y) + g(Y,V)\eta(X) - 2\eta(V)g(X,Y)\right],$$

for all vector fileds X, Y, V, where $\overline{\mathcal{L}}_V g$ is the Lie derivative in direction vector field V with respect to the Schouten-van Kampen connection,

$$(\overline{\mathcal{L}}_V g)(X,Y) := g(\overline{\nabla}_X V, Y) + g(X, \overline{\nabla}_Y V).$$

We define the generalized η-Ricci soliton with respect to the Schouten-van Kampen connection as follows

$$\alpha\bar{S} + \frac{\beta}{2}\overline{\mathcal{L}}_V g + \mu V^\flat \otimes V^\flat + \rho\eta\otimes\eta + \lambda g = 0, \tag{8}$$

where $\bar{S}$ is the Ricci tensor of the connection $\bar{\nabla}$, $V^\flat$ denotes the canonical 1-form associated to V that is $V^\flat(X) = g(V,X)$ for all vector field X, λ is a smooth function on M, and α, β, μ, ρ are real constants such that $(\alpha,\beta,\mu) \neq (0,0,0)$.

The generalized η-Ricci soliton equation becomes

(1) the η-Ricci soliton equation when $\alpha = 1$ and $\mu = 0$,

(2) the Ricci soliton equation when $\alpha = 1$, $\mu = 0$, and $\rho = 0$,

(3) the generalized Ricci soliton equation when $\rho = 0$.

3. Main results and their proofs

An f-Kenmotsu manifold is said to be η-Einstein if its Ricci tensor S is of the form

$$S = ag + b\eta \otimes \eta,$$

where a and b are smooth functions on manifold. Now, we consider M is an f-Kenmotsu manifold and it satisfies the generalized η-Ricci soliton (8) associated to the Schouten-van Kampen connection. Let the potential vector field V be a pointwise collinear vector field with the structure vector field ξ, that is, $V = \theta\xi$ for some function θ on M. Using (1) we have

$$\begin{aligned}\overline{\mathcal{L}}_{\theta\xi} g(X,Y) &= g(\nabla_X \theta\xi, Y) + g(X, \nabla_Y \theta\xi) + 2\theta f\left(\eta(X)\eta(Y) - g(X,Y)\right)\\ &= X(\theta)\eta(Y) + Y(\theta)\eta(X),\end{aligned} \tag{9}$$

for all vector fields X, Y. By definition of canonical 1-form associated to the vector field ξ we get

$$\xi^\flat \otimes \xi^\flat(X,Y) = \eta(X)\eta(Y), \tag{10}$$

for all vector fields X, Y. Inserting $V = \theta\xi$, (7), (9), and (10) in (8) we arrive at

$$\begin{aligned}&\alpha\left(S(X,Y) + (2f^2 + \xi(f))g(X,Y) + \xi(f)\eta(X)\eta(Y)\right) + \frac{\beta}{2}X(\theta)\eta(Y)\\ &+\frac{\beta}{2}Y(\theta)\eta(X) + (\mu\theta^2 + \rho)\eta(X)\eta(Y) + \lambda g(X,Y) = 0,\end{aligned} \tag{11}$$

for all vector fields X, Y. We plug $Y = \xi$ in (11) and using (6) to obtain

$$\frac{\beta}{2}X(\theta) + \frac{\beta}{2}\xi(\theta)\eta(X) + (\mu\theta^2 + \rho + \lambda)\eta(X) = 0, \tag{12}$$

for any vector field X. Taking $X = \xi$ in the equation (12) gives

$$\beta\xi(\theta) = -(\mu\theta^2 + \rho + \lambda). \tag{13}$$

Applying (13) in (12), we conclude

$$\beta X(\theta) = -(\mu\theta^2 + \rho + \lambda)\eta(X),$$

which yields

$$\beta d\theta = -(\mu\theta^2 + \rho + \lambda)\eta. \tag{14}$$

Substituting (14) in (11), we deduce

$$\alpha\bar{S}(X,Y) = \lambda(-g(X,Y) + \eta(X)\eta(Y)), \tag{15}$$

for all vector fields X, Y, which implies $\alpha\bar{r} = -2\lambda$.

Therefore, this leads to the following:

Theorem 3.1. *Suppose that $(M, g, \varphi, \xi, \eta)$ is an f-Kenmotsu 3-dimensional manifold. If M admits a generalized η-Ricci soliton $(g, V, \alpha, \beta, \mu, \rho, \lambda)$ with respect to the Schouten-van Kampen connection such that $\alpha \neq 0$ and $V = \theta\xi$ for some smooth function θ on M, then M is an η-Einstein soliton and an η-Einstein manifold with respect to the Schouten-van Kampen connection.*

From (15) we also have the following:

Corollary 3.2. *Let $(M, g, \varphi, \xi, \eta)$ be an f-Kenmotsu 3-dimensional manifold. If M admits a generalized η-Ricci soliton $(g, V, \alpha, \beta, \mu, \rho, \lambda)$ with respect to the Schouten-van Kampen connection such that $V = \theta\xi$ for some smooth function θ on M, then $\alpha\bar{r} = -2\lambda$.*

Remark 3.3. *Now, let M be an η-Einstein f-Kenmotsu 3-dimensional manifold with respect to the Schouten-van Kampen connection and $V = \xi$, that is, $\bar{S} = ag + b\eta \otimes \eta$ for some functions a and b on M. If a and b are constants then manifold M satisfies a generalized η-Ricci soliton $(g, \xi, \alpha, \beta = 0, \mu = 0, -b\alpha, -a\alpha)$ with respect to the Schouten-van Kampen connection.*

Substituting (7) in (15) we get

$$S(X,Y) + (2f^2 + \xi(f))g(X,Y) + \xi(f)\eta(X)\eta(Y) = \lambda(-g(X,Y) + \eta(X)\eta(Y)), \tag{16}$$

for all vector fields X, Y. Applying (3) in (16) we obtain

$$(\frac{r}{2} + 3f^2 + 2\xi(f) + \lambda)(g(X,Y) - \eta(X)\eta(Y)) = 0, \tag{17}$$

for all vector fields X, Y. Using (17) implies that

$$\frac{r}{2} + 3f^2 + 2\xi(f) + \lambda = 0. \tag{18}$$

Thus we can state the following theorem:

Theorem 3.4. *Let M be an f-Kenmotsu 3-dimensional manifold and it satisfies the generalized η-Ricci soliton $(g, \xi, \alpha, \beta, \mu, \rho, \lambda)$ with respect to the Schouten-van Kampen connection such that $\alpha \neq 0$ then $\lambda = -(\frac{r}{2} + 3f^2 + 2\xi(f))$.*

Definition 3.5. *A vector field V is said to be a conformal Killing vector field if*

$$(\mathcal{L}_V g)(X,Y) = 2hg(X,Y),$$

for all vector fields X, Y, where h is some function on M. The conformal Killing vector field V is called

- *proper when h is not constant,*
- *homothetic vector field when h is a constant,*
- *Killing vector field when $h = 0$.*

Let vector field V is a conformal Killing vector field with respect to the Schouten-van Kampen connection and satisfies in $\overline{\mathcal{L}}_V g = 2hg$. By (7) and (8) we have

$$\alpha\bar{S}(X,Y) + \beta hg(X,Y) + \mu V^\flat(X)V^\flat(Y) + \rho\eta(X)\eta(Y) + \lambda g(X,Y) = 0. \tag{19}$$

for all vector fields X, Y. By inserting $Y = \xi$ in (19) we have

$$g(\beta h\xi + \mu\eta(V)V + \rho\xi + \lambda\xi, X) = 0.$$

Since X is arbitrary vector field, we get the following theorem.

Theorem 3.6. *If the metric g of an f-Kenmotsu 3-dimensional manifold satisfies the generalized η-Ricci soliton $(g, V, \alpha, \beta, \mu, \rho, \lambda)$ with respect to the Schouten-van Kampen connection where V is conformally Killing vector field, that is $\mathcal{L}_V g = 2hg$, then*

$$(\beta h + \rho + \lambda)\xi + \mu\eta(V)V = 0.$$

Definition 3.7. *A nonvanishing vector field V on pseudo-Riemannian manifold (M, g) is called torse-forming [40] if*

$$\nabla_X V = fX + \omega(X)V, \tag{20}$$

for all vector field X, where ∇ is the Levi-Civita connection of g, f is a smooth function and ω is a 1-form. The vector field V is called

- *concircular [12, 39] whenever in (20) the 1-form ω vanishes identically,*
- *concurrent [30, 41] if in (20) the 1-form ω vanishes identically and $f = 1$,*
- *parallel vector field if in (20) $f = \omega = 0$,*
- *torqued vector field [13] if in (20) $\omega(V) = 0$.*

Let $(g, V, \alpha, \beta, \mu, \rho, \lambda)$ be a generalized η-Ricci soliton on an f-Kenmotsu 3-dimensional manifold where V is a torse-forming vector filed with respect to the Schouten-van Kampen connection and satisfied in $\bar{\nabla}_X V = fX + \omega(X)V$. Then

$$\alpha\bar{S}(X,Y) + (\overline{\mathcal{L}}_V g)(X,Y) + \mu V^\flat(X)V^\flat(Y) + \rho\eta(X)\eta(Y) + \lambda g(X,Y) = 0, \tag{21}$$

for all vector fields X, Y. On the other hand,

$$(\overline{\mathcal{L}}_V g)(X,Y) = 2fg(X,Y) + \omega(X)g(V,Y) + \omega(Y)g(V,X), \tag{22}$$

for all vector fields X, Y. Applying (22) into (21) we obtain

$$\alpha\bar{S}(X,Y) + [\beta f + \lambda]\, g(X,Y) + \rho\eta(X)\eta(Y) + \frac{\beta}{2}\left[\omega(X)g(V,Y) + \omega(Y)g(V,X)\right] + \mu g(V,X)g(V,Y) = 0,$$

for all vector fields X, Y. We take contraction of the above equation over X and Y to obtain

$$\alpha\bar{r} + 3\,[\beta f + \lambda] + \rho + \beta\omega(V) + \mu|V|^2 = 0.$$

Therefore we have the following theorem.

Theorem 3.8. *If the metric g of an f-Kenmotsu 3-dimensional manifold satisfies the generalized η-Ricci soliton $(g, V, \alpha, \beta, \mu, \rho, \lambda)$ with respect to the Schouten-van Kampen connection, where V is torse-forming vector filed and satisfied in (20), then*

$$\lambda = -\frac{1}{3}\left[\alpha(r + 6f^2 + 4\xi(f)) + \rho + \beta\omega(V) + \mu|V|^2\right] - \beta f.$$

4. Example

In this section, we give an example of f-Kenmotsu 3-dimensional manifold with respect to the Schouten-van Kampen connection.

Example 4.1. *Let (x, y, z) be the standard coordinates in $\mathbf{R}^3$ and $M = \{(x,y,z) \in \mathbf{R}^3 | z \neq 0\}$. We consider the linearly independent vector fields*

$$e_1 = z^2\frac{\partial}{\partial x}, \qquad e_2 = z^2\frac{\partial}{\partial y}, \qquad e_3 = \frac{\partial}{\partial z}.$$

We define the metric g by

$$g(e_i, e_j) = 1 \text{ if } i = j \text{ and } g(e_i, e_j) = 0 \text{ if } i \neq j,$$

for $i, j \in \{1, 2, 3\}$. We define an almost contact structure (φ, ξ, η) on M by

$$\xi = e_3, \quad \eta(X) = g(X, e_3), \quad \varphi = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

for all vector field X. Note the relations $\varphi^2(X) = -X + \eta(X)\xi$, $\eta(\xi) = 1$, and $g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y)$ hold. Hence, $(M, \varphi, \xi, \eta, g)$ defines an almost contact structure on M. We have

$[,]$	e_1	e_2	e_3
e_1	0	0	$-\frac{2}{z}e_1$
e_2	0	0	$-\frac{2}{z}e_2$
e_3	$\frac{2}{z}e_1$	$\frac{2}{z}e_2$	0

The Levi-Civita connection ∇ of M is described by

$$\nabla_{e_i} e_j = \begin{pmatrix} \frac{2}{z}e_3 & 0 & -\frac{2}{z}e_1 \\ 0 & \frac{2}{z}e_3 & -\frac{2}{z}e_2 \\ 0 & 0 & 0 \end{pmatrix}.$$

We see that the structure (φ, ξ, η) satisfies the formula $\nabla_X\xi = f(X - \eta(X)\xi)$ for $f = -\frac{2}{z}$, thus (M, ϕ, ξ, η, g) becomes an f-Kenmotsu 3-dimensional manifold. Now, using (5) we get the Schouten-van- Kampen connection on M as $\overline{\nabla}_{e_i} e_j = 0$ for $1 \leq i, j \leq 3$. Therefore $\bar{S} = 0$. If we consider $V = \xi$ then $\overline{\mathcal{L}}_V g = 0$. Therefore $(g, \xi, \alpha, \beta, \mu, \rho = -\mu, \lambda = 0)$ is a generalized η-Ricci soliton on manifold M.

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