AUT Journal of Mathematics and Computing

AUT J. Math. Comput., 3(2) (2022) 253-261
DOI: 10.22060/AJMC.2022.21726.1105

Original Article

A streamline algorithm for stochastic user equilibrium in interdependent bi-modal network

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ABSTRACT: Considering the stochastic traffic networks one can follow an assignment procedure to estimate flows. However, the interdependent bi-modal assignment problems are solved just for deterministic status. To this gap, this paper extends a traffic assignment problem for an interdependent bi-modal network under Stochastic User Equilibrium (SUE) conditions. To solve this problem, a new algorithm is presented by combining a user equilibrium algorithm namely Streamline algorithm with a Logit model. In our algorithm, the interaction between private and public traffic flows is explicitly modeled and travel time for each mode is considered as a function of two-mode flows. Also, the origin-destination matrix was split between two modes based on the binomial Logit function. Some networks were considered to illustrate the performance and the accuracy of the proposed stochastic user equilibrium algorithm on the interdependent bi-modal networks. Numerical results showed that this algorithm provided reasonable solutions with high accuracy in a small computation time compared with the other user equilibrium (UE) algorithms.

Review History:

Received:15 August 2022
Accepted:29 August 2022
Available Online:01 September 2022

Keywords:

Interdependent bi-modal network
Stochastic user equilibrium
Streamline algorithm
Transportation systems
Traffic optimization

(Dedicated to Professor S. Mehdi Tashakkori Hashemi)

1. Introduction

Traffic assignment is an important problem with a great application in transportation networks and simulation software. Many intelligent transportation systems [1] also depend on this problem. In the classical assignment problem, the first Wardrop's principle or user equilibrium (UE) condition has been considered in the assignment procedure where the journey times on all routes are equal to or less than those which would be experienced by a single traveler on any unused route [10]. UE principle usually assumes that the road network user has complete knowledge about the network configuration such as congestion and travel time information. Due to stochastic phenomena in real road networks, many factors such as time and cost cannot be accurately measured. They can be estimated by statics studies. So, travelers usually have different perceptions about the network status and their route choice behavior is usually stochastic.

By developing Advanced Traveler Information Systems (ATIS) through the provision of traffic information, commuters compensate for their limited knowledge and thus make more reasonable travel choice decisions. However, it cannot completely overcome the stochastic features of traffic networks. Lo and Szeto [8], Yin and Yang [16], and Li et al. [6] adopted the Logit-based stochastic user equilibrium (SUE) principle to describe route choice behaviors of both classes of drivers, equipped and unequipped with ATIS. Their approaches were adaptable to real scenarios since equipped or unequipped drivers could not accurately compute route travel disutility by considering the existence of

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various random factors in the real world. Huang and Li [3] considered mixed equilibrium in assignment problems, where UE is used for drivers equipped with ATIS and SUE for other users. Kuang and Huang [4] investigated SUE for multi-user classes with elastic demand using a variation inequality(VI) problem. To obtain SUE flow, Sheffi [10] presented a method of successive averages (MSA). Also, Liu et al. [7] presented two algorithms to contain the method of successive weighted averages (MSWA) algorithm and self-regulated averaging (SRA) schemes for SUE assignment. These algorithms have higher speed convergence than MSA. MSWA includes a new step size sequence, which gives higher weights to the auxiliary flow patterns and decreases the weights iteratively. In SRA scheme, step sizes are varying depending on the distance between the intermediate solution and auxiliary one.

Sometimes, imprecise parameters such as O-D demand, travel time, travel cost, and link capacity are given in uncertain conditions under stochastic or fuzzy attitude. Soudmand et al. [13] investigated congestion pricing under fuzzy condition. In this case, the lower level problem is a fuzzy assignment model with fuzzy link costs. Applying a famous defuzzification function, a real-valued multi-commodity flow problem can be obtained. They proposed a polynomial time interior point algorithm to find the optimal solution in assignment models. See also [15] for an approximation algorithm.

On the other hand, due to traffic congestion, air pollution, energy consumption, and road accidents of private transportation, we need to encourage users to public transportation. By considering public transportation as the alternative mode, two states can be considered for a combined network. First, a full-scale network with independent public links and private links can be considered. In the second state, the independence assumption is ignored to transfer private vehicles and public buses and two modes move jointly in the traffic stream. In the bi-modal traffic assignment model, important problems are the demand split between two modes and in the end establishing equilibrium for two modes. These problems are different in interdependent and independent bi-modal networks. There are few studies on the bi-modality network in transportation problems, see e.g.,[14, 5, 2].

Sheffi [10] investigated a bi-modal network assignment problem under UE condition by two states. In the first state, the super network reduces to several models which are solved jointly for finding UE flow patterns on both networks. In the second state, the super network considers the link interactions among dummy links and the basic network links. In this case, he used Streamline algorithm to find the UE flow patterns on interdependent bi-modal networks. Also, Miandoabchi et al. [9] considered network design problem for a bi-modal discrete urban road network with bus and car modes. In this problem, the interaction of automobile and bus flows is explicitly considered and a modal split assignment model is used to obtain the automobile and bus flows in the deterministic user equilibrium state. Si et al. [11] examined the assignment problem in a multi-modal network by considering the third walking mode.

Note that, the main factors that influence travelers' choices are flow split and assignment problem. Then, a bi-level model can be proposed for system optimization on urban roads. Moreover, Si et al. [12] presented a demand-based assignment model for the multi-modal and multiuser transportation systems. They analyzed the structural characteristics of urban multi-modal transport system through a two- layer network containing the traveler mode choice behavior as the first layer and vehicle routing as the second layer. The interferences between different vehicle flows are considered to find bi-equilibrium patterns for the multi-modal transport network. In all these works, flow assignment in a bi-modal network has been examined under UE conditions in both independent and interdependent cases.

By considering the SUE assignment, Ying and Yang [16] presented a general computational method for sensitivity analysis in the independent bi-modal network. Note that, the public network had no interference with the private vehicle network. They considered the Logit model for splitting demand between modes and completed SUE conditions in the combined transportation system. Then they solved the equilibrium traffic flow assignment problem with a unique solution.

In some networks, the public mode moves with the general private traffic and experiences the same congestion and delays. Also, the traverse time of each mode depends on the congestion level of the two modes. So, in this case, interdependent bi-modal networks must be considered. Based on the best of our knowledge, there are no published studies illustrating SUE in these networks.

This paper examines an SUE assignment problem in the second case of the bi-modal traffic network and considers interference between two modes. To solve this problem, a new algorithm is presented for the SUE problem in the assumed network combining the UE algorithm in the interdependent bi-modal network, Streamline algorithm, and Logit model.

The paper is organized as follows. In Section 2, a new traffic assignment model is approached. Section 3 describes basic steps of Streamline algorithm and presents a new algorithm for the assignment problem under SUE condition in the interdependent bi-modal network. In Section 4, some networks are used to explore the effectiveness and convergence of the proposed algorithm. Section 5 concludes the paper and provides recommendations for future works.

2. Problem definition and notations

We define a directed bi-modal transportation network with $G = (N, A)$, in which N and A correspond to the sets of nodes and links, respectively, W is the set of origin-destination pairs (O-D matrix) and $\overline{q}_{\mathrm{rs}}$ is a fixed demand between each O-D pair(r, s) that expresses the number of passengers who want to travel between the origin r and destination s. C_a is the capacity of link a that is given in terms of the vehicle per hour and R_{rs} refers to the set of all paths connecting O-D pair (r, s).

To model the interdependent bi-modal traffic assignment problem, at first, a super network must be constructed from the main network by transmuting each flow direction in two links containing private and public links. Therefore, two dummy networks of private and public resulted.

In the private network, A, W and R_{rs} are defined directly. q_{rs} is a private elastic demand, $\mathrm{h}_k^{\mathrm{rs}}$ and x_a are private flows on path $k \in R_{\mathrm{rs}}$ and link $a \in A$, respectively. t_a is a differentiable private performance function and t_{a0} is free flow travel time for the private vehicle of link a. c_k^{rs} is a total private cost for traveling on path $k \in R_{\mathrm{rs}}$ containing monetary time and toll. $\delta_{a,k}^{\mathrm{rs}}$ is a binary parameter, where $\delta_{a,k}^{\mathrm{rs}}$ is 1 if the link a exists in path k. Finally, U is the private vehicle occupancy in terms of passenger per vehicle. These parameters are presented in Table 1.

Table 1: Notation in private network

Notation	Description
q_{rs}	Elastic private O-D demand
h_k^{rs}	Private flow on path $k \in R_{rs}$
x_a	Private flow for link $a \in A$
t_{a0}	Free flow travel time on link a of private vehicle
t_a	Differentiable performance function on link a for private vehicle
c_k^{rs}	Total private cost of traveling on path $k \in R_{rs}$
U	vehicle occupancy in terms of passenger per vehicle
$\delta_{a,k}^{rs}$	$\begin{cases} 1 & \text{if link } a \text{ is on path } k \text{ from } r \text{ to } s \\ 0 & \text{otherwise} \end{cases}$

Parameters of Table 1 are expanded for public mode by using a hat (ˆ) over any variable (e.g., $\widehat{q}_{\mathrm{rs}}$, $\widehat{R}_{\mathrm{rs}}$, $\widehat{x}_a$, $\widehat{t}_a$, ...). Private and public performance functions are stated in Equations (1) and (2). Link's travel time for each mode is a function of two-mode flows. The interference between two modes is not certainly symmetric.

$$t_a = t_a\left(x_a,\ \widehat{x}_a\right) \qquad \forall\ a \in A,\ \forall\ a \in \widehat{A} \tag{1}$$

$$\widehat{t}_a = \hat{t}_a(\hat{x}_a,\ x_a) \qquad \forall\ a \in \hat{A},\ \forall\ a \in A \tag{2}$$

According to Sheffi's investigation [10], in an interdependent bi-modal network, because of non-symmetric features in the link performance function, there is no known mathematical programming that leads to UE flow patterns. But, if the Jacobian matrix of link-performance function is positive definite, the optimal solution can be obtained by using a direct solution method. Since this paper considers an interdependent bi-modal network, the performance function of the two modes was non-symmetric as well. Similar to UE statements, no equivalent minimization program can be formulated and no minimization algorithm can be applied for obtaining SUE flow patterns. Instead, a direct solution algorithm can be used for obtaining SUE flows in an interdependent bi-modal network. In the next section, details of the direct solution method are explained. Then by using the UE algorithm, a new Streamline algorithm will be presented for SUE assignment in an interdependent bi-modal network.

3. Direct solution method for SUE in interdependent bi-modal network

The important step in the Streamline algorithm is diagonalization, which works based on fixing all cross-link effects on link performance functions. It is used to obtain standard SUE flows and auxiliary flow patterns. Therefore, in the n^{th} iteration, by fixing $\widehat{x}^n$ in t_a , $\forall a \in A$, and x^n in $\widehat{t}_a$, $\forall a \in \widehat{A}$, the performance function of two modes are transformed to a single-variable function as $t_a = t_a\left(\omega,\ \widehat{x}^n\right)$ and $\widehat{t}_a = \widehat{t}_a(x^n, \omega)$ with variable ω. By substituting these single-variable functions in interdependent bi-modal SUE assignment, independent SUE has resulted which can be used to obtain auxiliary flow patterns. Now, new SUE flow patterns in the interdependent network can be stated as a convex combination of the old and auxiliary solutions.

3.1. SUE assignment in independent bi-modal network

At this point, SUE assignment in independent bi-modal will be briefly explained by applying the sensitivity analysis method [16]. To present the method, one can consider the private mode parameters of Table 2.

Table 2: Private mode parameters

Parameter	Description
S_{rs}	The expected minimum travel cost for O-D pair (r, s)
p_k^{rs}	Choice probability of path $k \in R_{rs}$ for private users
θ	The dispersion parameter in SUE assignment of private users
ψ_{rs}	The preference factor of private mode

Parameters in Table 2 can be also expanded for public mode.

To obtain complete SUE condition in an independent combined transportation system, we consider two standard SUE systems (3) and (4), simultaneously. These systems are used for private and public mods, where ∇ is the gradient operator and T denotes transposed matrix.

$$f(t,\hat{t}) = x - \sum_{rs} q_{rs}(\nabla_t S_{rs}\left(c^{rs}(t)\right))^T = 0 \tag{3}$$

$$\hat{f}(\hat{t},t) = \hat{x} - \sum_{rs} \hat{q}_{rs}(\nabla_t \hat{S}_{rs}(\hat{c}^{rs}(\hat{t})))^T = 0 \tag{4}$$

Two algorithms are used to find SUE equilibrium flows from (3) and (4) simultaneously. In these algorithms "demand split procedure" uses the Logit model and then assigns flows to each mode network. This procedure does the following steps:

1. Consider S_{rs} and $\widehat{S}_{\mathrm{rs}}$ are expected disutility functions for the private and public modes. They meet:

$$S_{\mathrm{rs}}\left(c^{\mathrm{rs}}(t)\right) = -\frac{1}{\theta} \ln \sum_k \exp\ \left(-\theta c_k^{\mathrm{rs}}\right) \qquad \forall\ \ (r,s) \in W \tag{5}$$

$$\widehat{S}_{\mathrm{rs}}\left(\widehat{c}^{\mathrm{rs}}\left(\widehat{t}\right)\right) = -\frac{1}{\widehat{\theta}} \ln \sum_k \exp\left(-\widehat{\theta}\widehat{c}_k^{\mathrm{rs}}\right) \qquad \forall\ \ (r,s) \in W \tag{6}$$

2. Total demand $\overline{q}_{\mathrm{rs}}$ is split between two modes according to the Logit model. Equations (7), and (8) make private and public demand $(q_{\mathrm{rs}}, \widehat{q}_{\mathrm{rs}})$:

$$q_{\mathrm{rs}} = D\left(S_{\mathrm{rs}}, \widehat{S}_{\mathrm{rs}}\right) = \overline{q}_{\mathrm{rs}} \frac{1}{1+\exp\ \left(-\alpha(\widehat{S}_{\mathrm{rs}} - S_{\mathrm{rs}} - \psi_{\mathrm{rs}})\right)} \qquad \forall (r,s) \in M \tag{7}$$

$$\widehat{q}_{\mathrm{rs}} = \overline{q}_{\mathrm{rs}} - q_{\mathrm{rs}} \tag{8}$$

Where α reflects characteristics of the traveler's behavior, regarding travel mode choice.

3.2. SUE Streamline algorithm in interdependent bi-modal network

This algorithm includes the following steps:

Step 1: Initialization:

Find a feasible link-flow vector $(x^n,\ \widehat{x}^n)$ by using free flow travel time $(t_0, \widehat{t}_0)$ using Equations (3) and (4). Set $n = 1$, γ_0=0, the real number $d \geq 0$, and the stopping criterion ϵ>0.

Step 2: Parameter tuning:

2.1- Travel time update: Set $t_a^n\left(x_a^n,\ \widehat{x}_a^n\right)\ \ , \forall a \in A$ and $\widehat{t}_a^n\left(\widehat{x}_a^n,\ x_a^n\right)\ \ , \forall a \in \widehat{A}$

2.2- Direction finding: Find an auxiliary flow pattern $(y^n,\ \widehat{y}^n)$ as:

- Fix $\widehat{x}_a^n$ in t_a, $(\forall a \in A)$ and also fix x_a^n in $\widehat{t}_a$, $(\forall a \in \widehat{A})$. Evaluate the one-vaiable function $t_a = t_a\left(\omega,\ \widehat{x}_a^n\right)$ and $\widehat{t}_a = \widehat{t}_a(x_a^n, \omega)$ with respect to ω. Solve standard SUE for two networks by solving Equations (3) and (4).

- Find split demand $(q_{\mathrm{rs}},\ \widehat{q}_{\mathrm{rs}})$ according to Logit model apply Equations (5), (6), (7) and (8) with respect to $t_a\left(\omega,\ \widehat{x}_a^n\right), \widehat{t}_a(x_a^n, \omega)$.

- Assign $(q_{\mathrm{rs}},\ \widehat{q}_{\mathrm{rs}})$ according to MSWA algorithm to each network to attain auxiliary flow pattern $(y^n,\ \widehat{y}^n)$.

Step 3: Move size determination (α_n) **and new flow pattern** $(x^{n+1}, \widehat{x}^{n+1})$:

- Since evaluating of SUE objective function is hard, finding the step size of the gradient-based algorithm for the UE assignment cannot be used. Instead, the MSWA algorithm applies a regular descending process for updating move size α_n. Similarly, we define α_n
- Thus, a new solution can be determined as follows:

$$x^{n+1} = x^n + \alpha_n \left(y^n - x^n\right) \tag{9}$$

$$\widehat{x}^{n+1} = \widehat{x}^n + \alpha_n \left(\widehat{y}^n - \widehat{x}^n\right) \tag{10}$$

Step 4: If convergence criterion happens, stop. Otherwise, set $n = n + 1$ and go to step 2.

- Convergence criterion can be defined as the following:

$$\frac{\sqrt{\sum_{a\in A}\left(x_a^{n+1} - x_a^n\right)^2}}{\sum_{a\in A} x_a^n} + \frac{\sqrt{\sum_{a\in \widehat{A}}\left(\widehat{x}_a^{n+1} - \widehat{x}_a^n\right)^2}}{\sum_{a\in \widehat{A}} \widehat{x}_a^n} < \epsilon \tag{11}$$

4. Example

Consider a simplified road network from a small town, consisting of 16 nodes, 40 O-D pairs, and 50 directed links. There are two mode choices for traveling: private and public. Also, there is an interaction between the two modes that leads to an interdependent bi-modal network depicted in Figure 1. Input parameters such as public free flow travel time (tG_a^0), private free flow travel time (tT_a^0) and capacity (C_a) for each link $a \in A$ are shown in Table 3. Table 4 presents O-D demand between 40 O-D pairs. BPR travel time functions (12) and (13) are considered for each virtual link of private and public networks. Other parameters containing dispersion parameters of two modes in SUE model $(\theta, \widehat{\theta})$, average occupant factor for private and public vehicles $\left(U, \widehat{U}\right)$, preference factor of private and public modes$\left(\psi_{\mathrm{rs}}, \widehat{\psi}_{\mathrm{rs}}\right)$ and reflecting parameter of the travelers' behavior characteristics in mode choice (α) are presented in Table 5.

$$t_a\left(x_a, \widehat{x}_a\right) = \mathrm{tT}_a^0 \left(1 + 0.15\left(\frac{\frac{x_a}{U} + \frac{\widehat{x}_a}{\widehat{U}}}{C_a}\right)^4\right) \qquad \forall a \in A, a \in \widehat{A} \tag{12}$$

$$\widehat{t}_a\left(\widehat{x}_a, x_a\right) = \mathrm{tG}_a^0 \left(1 + 0.15\left(\frac{\frac{\widehat{x}_a}{\widehat{U}} + \frac{x_a}{U}}{C_a}\right)^4\right) \qquad \forall a \in \widehat{A},\ a \in A \tag{13}$$

To argue the accuracy of the presented algorithm for the SUE assignment, simulation results were examined for some special cases of the considered network. The first case shows the difference in summation of optimal flows from all links between interdependent and independent bi-modal network assignment under SUE. To determine the solution pattern, first, this measure was calculated for UE assignment under different α according to the presented algorithms in [8]. Simultaneously, it was computed for the SUE assignment. Moreover, having more examinations was considered for different $(\theta, \widehat{\theta})$. Simulation data shows that, in comparison with UE, the variation process of the mentioned algorithm is correctly treated under the SUE assignment. Figure 2 shows this examination.

According to Figure 2, similar to the UE model, the closer α to number 1, the less the difference value for SUE with different $(\theta, \widehat{\theta})$. Finally, for $\theta = 1$, $\widehat{\theta} = 1$ and $\alpha = 1$, optimal values of SUE were close to UE.

Also, two dispersion parameters θ and $\widehat{\theta}$ play an important role in the assignment process.To display effects of $(\theta, \widehat{\theta})$ on optimal flow and reliability, the optimal flow from the proposed algorithm under different $(\theta, \widehat{\theta})$ and also optimal flow of the UE algorithm in the interdependent bi-modal network was determined. Then, the maximum difference of flows between SUE and UE for all links under each $(\theta, \widehat{\theta})$ was computed.

Figure 3 shows this comparison concerning to difference$(\theta, \widehat{\theta})$. Comparison results demonstrate the closeness of UE and SUE in interdependent bi-modal networks.

According to Figure 3, similar to the independent state, in the interdependent case, the closer $(\theta, \widehat{\theta})$ to 1, the less the maximum difference flow between SUE and UE. Besides the maximum difference between interdependent and independent networks are small for all $(\theta, \widehat{\theta})$.

One of the most important criteria in the evaluation of assignment algorithms is their convergence to optimal flow. For this purpose, Figure 4 presents run-times for the presented SUE algorithm and UE under different

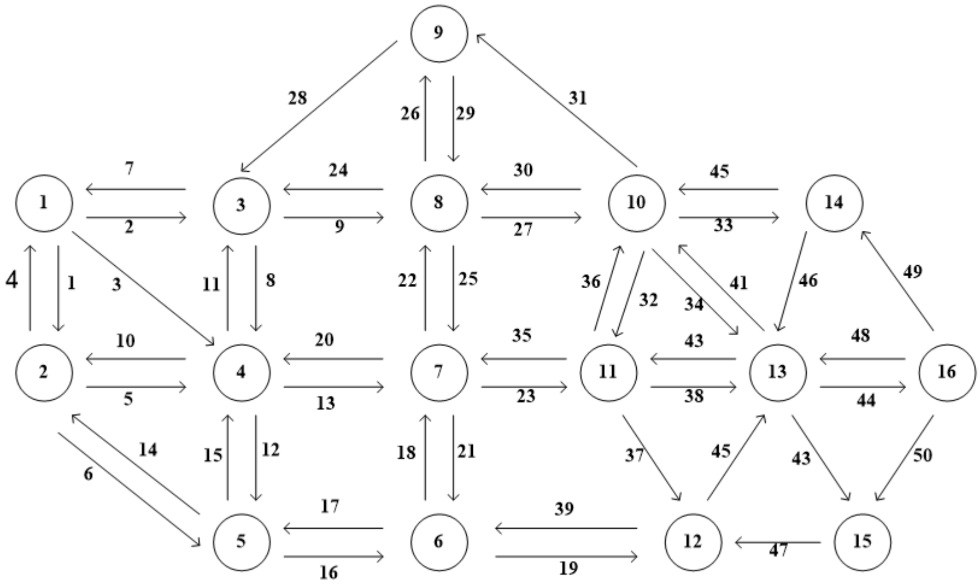

Figure 1: Example network with 16 nodes, 40 O-D pairs and 50 directed links.

Table 3: Input parameters containing free flow travel times of two modes and capacity for each link

Link	tT_a^0	tG_a^0	C_a	Link	tT_a^0	tG_a^0	C_a
1	**2.6**	**2.7**	**367**	**26**	**2.5**	**3**	**838**
2	**2.98**	**3.1**	**475**	**27**	**1.85**	**1.95**	**950**
3	**3.23**	**3.4**	**256**	**28**	**3.83**	**4**	**701**
4	**2.57**	**2.75**	**350**	**29**	**2.51**	**3**	**866**
5	**3.29**	**3.4**	**955**	**30**	**1.9**	**2**	**631**
6	**4**	**4.1**	**576**	**31**	**4.56**	**4.7**	**726**
7	**3.08**	**3.18**	**490**	**32**	**1.5**	**1.7**	**638**
8	**3.2**	**3.4**	**618**	**33**	**2.11**	**2.2**	**785**
9	**3.1**	**3.2**	**655**	**34**	**4.12**	**4.2**	**778**
10	**3.28**	**3.45**	**980**	**35**	**1.62**	**1.8**	**880**
11	**3.15**	**3.3**	**619**	**36**	**1.52**	**1.65**	**619**
12	**3.86**	**4**	**524**	**37**	**0.15**	**0.2**	**275**
13	**2.15**	**2.3**	**886**	**38**	**2.02**	**2.15**	**805**
14	**4.12**	**4.2**	**544**	**39**	**2.78**	**2.85**	**822**
15	**3.85**	**4**	**738**	**40**	**2.15**	**2.3**	**875**
16	**3.8**	**3.95**	**900**	**41**	**2.06**	**2.18**	**806**
17	**3.75**	**3.85**	**900**	**42**	**2.05**	**2.15**	**819**
18	**2.02**	**2.1**	**701**	**43**	**2.22**	**2.35**	**907**
19	**2.77**	**2.9**	**722**	**44**	**2.21**	**2.3**	**998**
20	**2.12**	**2.25**	**879**	**45**	**3.18**	**3.3**	**799**
21	**1.27**	**1.4**	**830**	**46**	**1.58**	**1.7**	**655**
22	**1.51**	**2**	**854**	**47**	**2.05**	**2.17**	**738**
23	**1.65**	**1.85**	**890**	**48**	**2.38**	**2.5**	**894**
24	**3.75**	**3.9**	**652**	**49**	**2.06**	**2.15**	**982**
25	**1.47**	**1.6**	**664**	**50**	**3.25**	**3.4**	**757**

convergence accuracies. Note that, Logarithm of run-times was considered because of their magnitude. In this Figure the convergence speed with respect to different stopping criterion (ϵ) for the proposed SUE algorithm and UE in interdependent bi-modal network were presented.

This Figure shows that decreasing the stopping criterion (ϵ) increases the run-time for both SUE and UE. However, the increment for SUE was very smaller than UE. Even for extremely small values of ϵ UE did not converges, while SUE converges rapidly. Finally, the presented SUE algorithm was faster than UE in the interdependent bi-modal network. Thus the proposed algorithm for SUE in the bi-modal interdependent network could find optimal flow with acceptable error and processing time.

5. Conclusion

This paper models a traffic network as an interdependent bi-modal network including public and private modes. To match real traffic conditions, stochastic properties of traffic networks are considered using the SUE assignment model. We proposed a Streamline algorithm for SUE in the considered network. The following conclusions were drawn from our study:

Table 4: O-D matrix between 40 origin-destinations

O-D pair	Demand	O.D pair	Demand
(1,7)	225	(9,5)	200
(1,12)	200	(9,7)	230
(1,13)	250	(9,12)	188
(1,16)	180	(9,13)	150
(2,7)	150	(9,16)	175
(2,9)	190	(12,1)	250
(2,12)	196	(12,2)	108
(2,13)	200	(12,9)	220
(2,14)	210	(13,1)	155
(2,16)	230	(13,2)	185
(5,9)	300	(13,5)	195
(5,13)	250	(13,9)	130
(5,14)	250	(14,2)	196
(5,16)	225	(14,5)	283
(7,1)	279	(14,7)	188
(7,2)	230	(16,1)	159
(7,9)	250	(16,2)	238
(7,14)	250	(16,5)	136
(7,16)	350	(16,7)	192
(9,2)	275	(16,9)	185

Table 5: Some parameters for SUE assignment in interdependent bi-modal network

θ	$\widehat{\theta}$	U	$\hat{U}$	ψ_{rs}	$\hat{\psi}_{rs}$	α
0.5	0.5	1	10	10	1	0.5

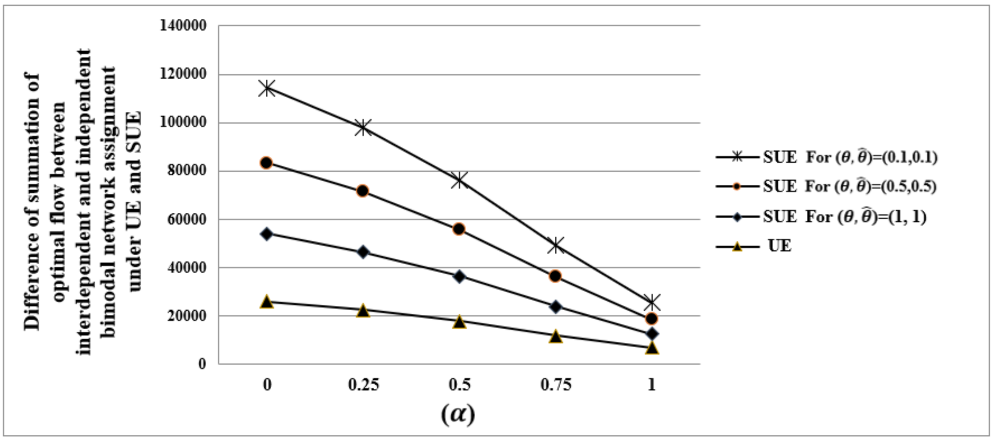

Figure 2: Difference summation of optimal flow between interdependent and independent bi-modal network assignment problems under SUE and UE.

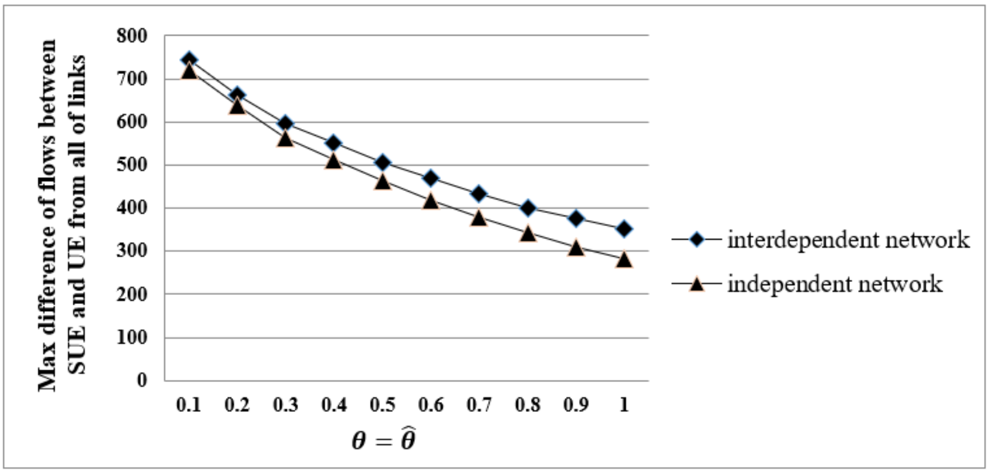

Figure 3: Maximum difference between SUE and UE flows in interdependent and independent bi-modal network.

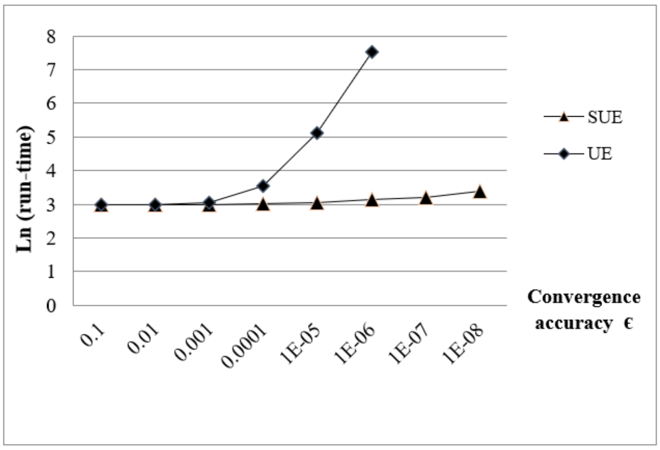

Figure 4: SUE and UE run-time under different convergence accuracies ϵ.

- For any dispersion parameter $(\theta, \widehat{\theta})$ and different reflecting parameter α, the behavioral pattern of the proposed SUE was close to the same patterns of UE model.
- For $\theta = 1, \widehat{\theta} = 1$ and $\alpha = 1$, the optimal values of SUE and UE were similar.
- Comparison results showed that mutation from UE to SUE in an interdependent bi-modal network using the proposed algorithm was applicable in real cases.
- The convergence time and accuracy of our SUE-based model are better than those of UE models.
- The convergence speed of our algorithm was much better than the UE model under different stopping criterion ϵ.
- For some small values ϵ, UE could not converge to the optimal solution, while SUE converges in a few times.

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Please cite this article using:

Somayeh Soudmand, Mahshid Falsafi, A streamline algorithm for stochastic user equilibrium in interdependent bi-modal network, AUT J. Math. Comput., 3(2) (2022) 253-261
DOI: 10.22060/AJMC.2022.21726.1105