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Original Article

A new MILP model for vehicle routing-loading problem under fragility, LIFO, and rotation constraints

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ABSTRACT: Simultaneous optimization of vehicle routing and loading decisions in three-dimensional case is one of the important problems in logistics and has received great attention from researchers. To the best of our knowledge, optimization models presented in the literature for this problem either are too complicated or do not include important loading assumptions such as item fragility, last-in-first-out arrangement, and the possibility of rotation. To overcome the shortcoming of the existing models, in this paper, we present a novel mixed-integer linear programming (MILP) model which not only involves important loading assumptions, but also does not have the complexity of previous models. Moreover, we provide valid inequalities to strengthen the LP relaxation bound and accelerate the solution process. Further, we show that how a restricted version of our model can be incorporated in loading procedures of meta-heuristic algorithms to improve their efficiency. Computational results over instances, taken from the literature, show the performance of the proposed approach.

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1. Introduction

The growth of global population, the evolution of human societies' interaction, the development of trade exchange and online shopping, and the increasing concerns about environmental pollution have made the well-known vehicle routing problem (VRP) even more critical. Specially, the pandemic Corona virus disease has spurred the demand in facilities dealing with VRP such as online stores and take-out food restaurants. So far, many studies have been conducted on different variants of VRP such as capacitated VRP (CVRP) in which the capacity of the vehicle container is assumed to be limited, VRP with time-window [3], VRP with pick-up and delivery [14], green VRP [1], time-dependent VRP [13], etc. The interested reader is referred to [7] and [10] for a comprehensive overview.

Together with routing decisions, the placement of customers' demands inside the vehicle container is also an important aspect, and neglecting it may disrupt the plan and impose additional costs. Hence, vehicle routing and loading problem (VRLP), as one of the most important variant of CVRP, has recently received great attention.

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In this problem, both of the routing and loading decisions are simultaneously decided within a single optimization problem considering different loading aspects such as vehicles' container capacity, fragility of items, rotation, and last-in-first-out (LIFO) arrangement. As pointed out by Bortfeldt and Wäscher [6], VRLP can be divided into two-dimensional and three-dimensional categories which we refer to as 2VRLP and 3VRLP, respectively, for short. 2VRLP considers customers' demands and the vehicle container in two dimensional case (i.e., just the length and width are taken into account and height information is ignored). It is applicable when items cannot be stacked on top of each other due to their weight, fragility or high height. For some related works, see [15, 21, 32, 20, 22, 9, 17]. In 3VRLP which is the focus of this paper, customers' demands and the vehicle container are considered in three dimensional case. In the continuation of this section, first, some concepts related to 3VRLP are described, then, the relevant literature is reviewed and the innovations of this paper are discussed.

1.1. Preliminary concepts

In 3VRLP, a set of customers must be visited by a fleet of homogeneous or heterogeneous vehicles stationed at the depot. It is assumed that the demands of each customer are in the form of multiple (or single) rectangular-cube boxes with specific weight, length, width and height. Throughout the paper, we may use the term "box" when referring to customers' demands. 3VRLP aims at partitioning customers into at most V routes, one for each vehicle, so that the vehicle leaves the depot, visits the assigned customers, and then, returns to the depot, and all boxes demanded by each customer are delivered by a single vehicle and in a single visit. In addition to routing decisions, the placement of boxes inside the vehicles' container should also be decided and the loading restrictions should be satisfied. In what follows, some of the essential and common loading constraints are addressed:

- Non-overlapping: This constraint indicates that boxes placed in the same vehicle cannot overlap each other and should be entirely put in the vehicle container.
- Connectivity: This constraint implies that all boxes, demanded by any given customer, should be delivered by a single vehicle.
- Weight-capacity: This constraint indicates that the total weight of boxes delivered by a given vehicle cannot exceed its weight-capacity.
- Orthogonality: This constraint indicates that boxes should be placed in the vehicle container so that their edges are parallel to the container edges.
- Rotation: Rotation of boxes may be prohibited or allowed. For example, "fixed vertical orientation" is a common assumption indicating that vertical axes of boxes should be parallel to the vertical axes of the container, while a 90-degree rotation is allowed on the horizontal plane.
- Fragility restriction: This constraint is stated for boxes containing fragile goods and having low resilience and indicates that non-fragile boxes cannot be placed on fragile ones; however, a fragile box can be stacked on both fragile and non-fragile boxes.
- Stability: When a box is stacked on other boxes or placed directly on the container floor, its base should be supported by a minimum supporting area so that unwanted movement of boxes is prevented during travel. In practice, stability can also be satisfied by filling empty spaces between boxes via filler materials.
- LIFO constraint: This constraint ensures that when a given customer is visited, all of his/her boxes can be unloaded without repositioning of boxes of other subsequent customers and just by straight movements parallel to the container edges. According to this constraint, the placement of boxes inside the container of any given vehicle is dependent on the order that vehicle visits the customers.

In the above description, we just focused on constraints that are more common in practice and more relevant to the ones considered in the subsequent sections. However, loading constraints are not limited to those mentioned above, and interested readers are referred to [6, 27] for more detail.

1.2. Literature review and our innovations

3VRLP was first addressed by Gendreau et al. [12] under LIFO and fragility constraints with fixed vertical orientation and the possibility of rotation in the horizontal plane. They proposed a tabu-search (TS) algorithm to solve the problem. Since then, various studies focused on developing heuristic and meta-heuristic algorithms to solve this NP-hard problem. For example, Bortfeldt and Homberger [5] proposed a two-phase "packing first, routing second" method for 3VRLP with time-window, LIFO and stability constraints. Moura and Oliveira [26] presented a hierarchical method for solving 3VRLP with time-window and LIFO constraints where, first, the routes are planned, and then for each route, the boxes are loaded into the vehicle container. Ceschia et al. [8] addressed 3VRLP with heterogeneous fleet under LIFO, fragility, fixed orientation and the possibility of delivery splitting, and solved it via a local-search heuristic method. Amongst different meta-heuristic methods proposed for 3VRLP, one can mention ant-colony algorithm [11], TS [30, 4, 29], honey bee mating optimization [28], genetic algorithm [23], etc. For a comprehensive overview on various solution methods developed for 3VRLP, see [27, 19].

Despite extensive research conducted on developing heuristic and meta-heuristic algorithms for 3VRLP, to the best of our knowledge, only a few papers have presented mathematical optimization models for 3VRLP with practical loading restrictions such as LIFO, fragility and the possibility of rotation. Moura and Oliveira [26] formulated a mixed-integer linear programming (MILP) model for 3VRLP with weight-capacity and time-window constraints under the assumption of the possibility of rotation. However, their model does not include fragility and LIFO constraints. Junqueira et al. [16] and Vega-Mejía et al. [31] assumed that the rotation is not allowed, and presented a MILP model satisfying fragility, LIFO and stability constraints. Considering a Cartesian coordinate system for the vehicle's container, their model requires a binary variable with indices (i, v, b, t, x, y, z) that is one if customer i is visited in the tth order of the route assigned to vehicle v and the left-back-bottom corner of the bth box demanded by him/her is placed on the point (x, y, z) of the Cartesian coordinate system associated with vehicle v; otherwise, zero. This leads to a huge number of binary variables so that even very small-sized instances of the model will not be solvable, and the model lacks the necessary efficiency due to its high complexity. Ruan et al. [28] have used a similar definition for variables with the difference that they have not studied routing and loading decisions simultaneously in a single optimization model. Instead, first a model is solved to plan the routes, and then another model is solved to place boxes in the vehicle's container. Moura [25] and Ayough et al. [2] proposed MILP models involving the possibility of rotation, weight-capacity, and time-window constraints; however, fragility and LIFO restrictions are not contained in their models.

To the best of our knowledge, no optimization model has been proposed for 3VRLP including the important constraints of fragility, LIFO and the possibility of rotation at the same time; and the models which contain some of these constraints are too complicated. Therefore, as the first contribution, in this paper, we present a new optimization model for 3VRLP, which involves the possibility of rotation, fragility and LIFO constraints, and does not have the complexity of previous models. Further, we propose valid inequalities and symmetry-breaking constraints to improve LP-relaxation bound and speed up the solution process of the model via MILP solvers. As another contribution, we show that how a restricted version of our model can be incorporated into the loading procedures of meta-heuristics to improve their efficiency in solving large-sized instances of 3VRLP. Computational experiments over a variety of instances, taken from the literature, confirm the effectiveness of the proposed approach.

The rest of this paper is organized as follows: In Section 2, the problem 3VRLP with the possibility of rotation under fragility and LIFO constraints is described in more detail. Then, it is formulated as a MILP model, and some valid inequalities and symmetry-breaking constraints are proposed to improve the formulation quality. Section 3 evaluates the model and the performance of valid inequalities. Section 4 describes the importance of the model and shows how it can improve the performance of the loading procedures of meta-heuristic algorithms. Finally, Section 5 concludes and offers directions for future research.

2. Problem description and formulation

In this section, first, the problem description, adopted from [29], is presented, and then the MILP model and valid inequalities are provided.

2.1. Problem description

Let $\mathbb{I} = \{1, 2, \ldots, n\}$ (indexed by i) be the set of customers and consider 0 as the depot, and put $\mathbb{I}_0 = \mathbb{I} \cup \{0\}$. For every $i, j \in \mathbb{I}_0$, assume that $d_{i,j}$ shows the distance between the points i and j. Let $\mathbb{V} = \{1, 2, \ldots, V\}$, indexed by v, be the set of homogenous vehicles with the weight-capacity g stationed at the depot assuming that the length, width and height of vehicles' containers are equal to L, W, and H, respectively. A three-dimensional coordinate system is used to show the container assuming that the origin of the system corresponds to left-back-bottom corner point of the container. For more illustration, see Figure 1.

Let $\mathbb{B}_i$, indexed by b, be the set of all boxes demanded by customer i; and for every $b \in \mathbb{B}_i$, assume that $g'_{i,b}$, $l_{i,b}$, $w_{i,b}$, and $h_{i,b}$, show the weight, length, width, and height of the bth box demanded by customer i, respectively. Further, let $f_{i,b}$ be a binary parameter that is 1 if the bth box, demanded by customer i, is fragile, otherwise 0. The demand of each customer should be totally satisfied by a single vehicle and in a single visit. In the placement of boxes inside the container, vertical orientation is assumed to be fixed while a 90-degree rotation is allowed on the horizontal plane. See Figure 2 for more illustration. Customers should be partitioned into at most V routes, one for each vehicle, so that the vehicle leaves the depot, visits the assigned customers, and then, returns to the

depot. The objective is to minimize the total traveled distance while satisfying the fragility, LIFO and rotation constraints. The following assumptions are made:
A1: Vehicles are assumed to be homogeneous.
A2: All boxes demanded by each customer should be delivered by a single vehicle and in a single visit.
A3: The weight-capacity of the vehicles' container cannot be violated.
A4: Boxes placed in the same vehicle cannot overlap each other.
A5: Vertical orientation is assumed to be fixed while a 90-degree rotation is allowed on the horizontal plane.
A6: The edges of the boxes placed in the same vehicle should be parallel to the container edges.
A7: Fragile boxes cannot be placed under non-fragile ones; however, a fragile box can be placed upon any box.
A8: LIFO constraint should be observed so that when a given customer is visited, all of his/her boxes are unloaded without repositioning of boxes of other subsequent customers and just by straight movements parallel to the container edges. To satisfy this, the placement of boxes inside the container of a given vehicle would be dependent on the order by which the vehicle visits the customers. That is if customers i and j belong to the same vehicle, and customer i is visited earlier than j, each box of customer j should be placed either to the right, or to the left, or back, or below of any box of customer i.

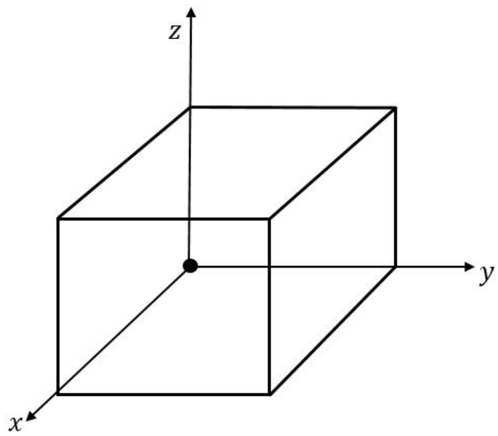

Figure 1: Three-dimensional coordinate system associated with the vehicle container

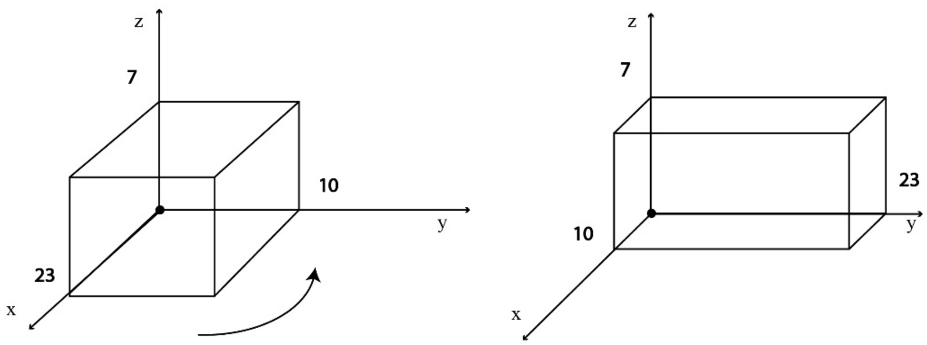

Figure 2: 90-degree rotation

2.2. MILP model

The following notations are defined:
Sets, indices, and parameters

$\mathbb{I} = \{1, 2, \ldots, n\}$: Set of customers (indexed by i, j, k)
$\mathbb{I}_0 = \mathbb{I} \cup \{0\}$: Set of points including customers and the depot
$\mathbb{V} = \{1, 2, ..., V\}$: Set of vehicles (indexed by v)
$\mathbb{B}_i = \{1, 2, \ldots, B_i\}$: Set of boxes demanded by customer i (indexed by b)
$d_{i,j}$: Distance between the points i and j
$t_{i,j}$: Travel time between the points i and j
L, W, H: Length, width and height of the container of each vehicle, respectively
$l_{i,b}, w_{i,b}, h_{i,b}$: Length, width and height of the bth box demanded by customer i, respectively
g: Weight-capacity of each vehicle
$g'_{i,b}$: Weight of the bth box demanded by customer i
$f_{i,b}$: Binary parameter that is 1 if the bth box demanded by customer i is fragile; otherwise 0
M_1, M_2, M_3, M': Sufficiently large positive numbers
ρ_1, ρ_2, ρ_3: Weights used in the objective function to ensure that the boxes are placed in each container as tightly as possible (i.e. the leftover space between boxes is minimized)
Decision variables
α_v: Binary variable that is 1 if vehicle v is used, otherwise 0 ($v \in \mathbb{V}$)
$\beta_{i,v}$: Binary variable that is 1 if customer i is assigned to vehicle v, otherwise, 0 ($i \in \mathbb{I}, v \in \mathbb{V}$)
$\delta_{i,j,v}$: Binary variable that is 1 if vehicle v travels directly from the point i to the point j, otherwise 0 ($i, j \in \mathbb{I}_0 : i \neq j, v \in \mathbb{V}$)
$\gamma_{i,j}$: Binary variable that is 1 if customers i and j are both assigned to the same vehicle and customer i is visited before customer j (not necessarily immediately), otherwise 0 ($i, j \in \mathbb{I} : i \neq j$)
$\theta_{i,b}$: Binary variable that is 1 if the bth box demanded by customer i is placed inside the container without any rotation in the horizontal plane, otherwise 0 ($i \in \mathbb{I}, b \in \mathbb{B}_i$)
$\eta_{i,b,j,b'}$: Binary variable that is 1 if the b'th box demanded by customer j is placed in front of the bth box demanded by customer i, otherwise 0 ($i, j \in \mathbb{I}, b \in \mathbb{B}_i, b' \in \mathbb{B}_j : ((i = j \wedge b \neq b') \vee (i \neq j))$)
$\eta'_{i,b,j,b'}$: Binary variable that is 1 if the b'th box demanded by customer j is placed in the right side of the bth box demanded by customer i, otherwise 0 ($i, j \in \mathbb{I}, b \in \mathbb{B}_i, b' \in \mathbb{B}_j : ((i = j \wedge b \neq b') \vee (i \neq j))$)
$\eta''_{i,b,j,b'}$: Binary variable that is 1 if the b'th box demanded by customer j is placed above the bth box demanded by customer i, otherwise 0 ($i, j \in \mathbb{I}, b \in \mathbb{B}_i, b' \in \mathbb{B}_j : ((i = j \wedge b \neq b') \vee (i \neq j))$)
q_i: Nonnegative continuous variable representing the time at which customer i is visited (if $i \in \mathbb{I}$), or shows the departure time at the depot (if $i = 0$)
$(x_{i,b}, y_{i,b}, z_{i,b})$: Nonnegative continuous variables indicating the coordinates at which the left-back-bottom corner of the bth box demanded by customer i is placed. We refer to the left-back-bottom corner point of the box b as the reference point. Note that the reference point is not predetermined in advance and depends on whether or not the box is horizontally rotated ($i \in \mathbb{I}, b \in \mathbb{B}_i$) With respect to above notations, the problem is formulated as the following MILP model which we refer to as 3VRLP.
$(3VRLP)$

$$\min \sum_{v\in\mathbb{V}} \sum_{i\in\mathbb{I}_0} \sum_{j\in\mathbb{I}_0} d_{i,j}\delta_{i,j,v} + \sum_{i\in\mathbb{I}} \sum_{b\in\mathbb{B}_i} (\rho_1 x_{i,b} + \rho_2 y_{i,b} + \rho_3 z_{i,b}) \tag{1}$$

$s.t.$

$$\sum_{v\in\mathbb{V}} \beta_{i,v} = 1 \qquad \forall i \in \mathbb{I} \tag{2}$$

$$\beta_{i,v} \leq \alpha_v \qquad \forall i \in \mathbb{I}, \forall v \in \mathbb{V} \tag{3}$$

$$\sum_{j\in\mathbb{I}} \delta_{0,j,v} = \alpha_v \qquad \forall v \in \mathbb{V} \tag{4}$$

$$\sum_{j\in\mathbb{I}} \delta_{j,0,v} = \alpha_v \qquad \forall v \in \mathbb{V} \tag{5}$$

$$\sum_{i\in\mathbb{I}_0 : i\neq j} \delta_{i,j,v} = \sum_{i\in\mathbb{I}_0 : i\neq j} \delta_{j,i,v} \qquad \forall v \in \mathbb{V}, \forall j \in \mathbb{I} \tag{6}$$

$$\sum_{i\in\mathbb{I}_0 : i\neq j} \delta_{i,j,v} = \beta_{j,v} \qquad \forall j \in \mathbb{I}, \forall v \in \mathbb{V} \tag{7}$$

$$\sum_{i\in\mathbb{I}} \beta_{i,v} \sum_{b\in\mathbb{B}_i} g'_{i,b} \leq g\alpha_v \qquad \forall v \in \mathbb{V} \tag{8}$$

$$q_j \geq q_i + t_{i,j} - M^{'}\left(1 - \sum_{v\in\mathbb{V}} \delta_{i,j,v}\right) \qquad \forall i,j \in \mathbb{I} : i \neq j \tag{9}$$

$$\delta_{i,j,v} \leq \gamma_{i,j} \qquad \forall i,j \in \mathbb{I} : i \neq j, \forall v \in \mathbb{V} \tag{10}$$

$$\gamma_{i,j} + \gamma_{j,k} \leq 1 + \gamma_{i,k} \qquad \forall i,j,k \in \mathbb{I} : i \neq j, i \neq k, j \neq k \tag{11}$$

$$\gamma_{i,j} + \gamma_{j,i} \leq 1 \qquad \forall i,j \in \mathbb{I} : i \neq j \tag{12}$$

$$\gamma_{i,j} + \gamma_{j,i} \leq \left(2 - \beta_{i,v} - \beta_{j,v'}\right) \qquad \forall i,j \in \mathbb{I} : i \neq j, \forall v, v^{'} \in \mathbb{V} : v \neq v^{'} \tag{13}$$

$$\gamma_{i,j} + \gamma_{j,i} \geq 1 - \left(2 - \beta_{i,v} - \beta_{j,v}\right) \qquad \forall i,j \in \mathbb{I} : i \neq j, \forall v \in \mathbb{V} \tag{14}$$

$$\eta_{i,b,j,b'} + \eta_{j,b',i,b} + \eta^{'}_{i,b,j,b'} + \eta^{'}_{j,b',i,b} + \eta^{''}_{i,b,j,b'} + \eta^{''}_{j,b',i,b} \leq 3\left(\gamma_{i,j} + \gamma_{j,i}\right) \qquad \forall i,j \in \mathbb{I} : i < j, \forall b \in \mathbb{B}_i,\ \forall b^{'} \in \mathbb{B}_j \tag{15}$$

$$\eta_{j,b',i,b} + \eta^{'}_{i,b,j,b'} + \eta^{'}_{j,b',i,b} + \eta^{''}_{j,b',i,b} \geq \gamma_{i,j} \qquad \forall i,j \in \mathbb{I} : i \neq j, \forall b \in \mathbb{B}_i,\ \forall b^{'} \in \mathbb{B}_j \tag{16}$$

$$\eta_{i,b,i,b'} + \eta_{i,b',i,b} + \eta^{'}_{i,b,i,b'} + \eta^{'}_{i,b',i,b} + \eta^{''}_{i,b,i,b'} + \eta^{''}_{i,b',i,b} \geq 1 \qquad \forall i \in \mathbb{I}, \forall b, b^{'} \in \mathbb{B}_i : b < b^{'} \tag{17}$$

$$\begin{gathered}\eta_{i,b,j,b'} + \eta_{j,b',i,b} + \eta^{'}_{i,b,j,b'} + \eta^{'}_{j,b',i,b} + \eta^{''}_{j,b',i,b} \geq 1 - \left(1 - \gamma_{i,j} - \gamma_{j,i}\right) \\ \forall i,j \in \mathbb{I}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : i \neq j, f_{i,b} = 1, f_{j,b'} = 0\end{gathered} \tag{18}$$

$$\eta_{i,b,i,b'} + \eta_{i,b',i,b} + \eta^{'}_{i,b,i,b'} + \eta^{'}_{i,b',i,b} + \eta^{''}_{i,b',i,b} \geq 1 \qquad \forall i \in \mathbb{I}, \forall b, b^{'} \in \mathbb{B}_i : b \neq b^{'},\ f_{i,b} = 1, f_{i,b'} = 0 \tag{19}$$

$$\begin{gathered}x_{i,b} + l_{i,b}\theta_{i,b} + w_{i,b}\left(1 - \theta_{i,b}\right) \leq x_{j,b'} + M_1\left(1 - \eta_{i,b,j,b'}\right) \\ \forall i,j \in \mathbb{I}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b'\right) \vee \left(i \neq j\right)\right)\end{gathered} \tag{20}$$

$$\begin{gathered}y_{i,b} + w_{i,b}\theta_{i,b} + l_{i,b}\left(1 - \theta_{i,b}\right) \leq y_{j,b'} + M_2\left(1 - \eta^{'}_{i,b,j,b'}\right) \\ \forall i,j \in \mathbb{I}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b'\right) \vee \left(i \neq j\right)\right)\end{gathered} \tag{21}$$

$$z_{i,b} + h_{i,b} \leq z_{j,b'} + M_3\left(1 - \eta^{''}_{i,b,j,b'}\right) \qquad \forall i,j \in \mathbb{I}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b^{'}\right) \vee \left(i \neq j\right)\right) \tag{22}$$

$$x_{i,b} + l_{i,b}\theta_{i,b} + w_{i,b}\left(1 - \theta_{i,b}\right) \leq L \qquad \forall i \in \mathbb{I}, \forall b \in \mathbb{B}_i \tag{23}$$

$$y_{i,b} + w_{i,b}\theta_{i,b} + l_{i,b}\left(1 - \theta_{i,b}\right) \leq W \qquad \forall i \in \mathbb{I}, \forall b \in \mathbb{B}_i \tag{24}$$

$$z_{i,b} + h_{i,b} \leq H \qquad \forall i \in \mathbb{I}, \forall b \in \mathbb{B}_i \tag{25}$$

$$\alpha_v \in \{0,1\} \qquad \forall v \in \mathbb{V} \tag{26}$$

$$\beta_{i,v} \in \{0,1\} \qquad \forall i \in \mathbb{I}, \forall v \in \mathbb{V} \tag{27}$$

$$\delta_{i,j,v} \in \{0,1\} \qquad \forall i,j \in \mathbb{I}_0 : i \neq j, \forall v \in \mathbb{V} \tag{28}$$

$$\gamma_{i,j} \in \{0,1\} \qquad \forall i,j \in \mathbb{I} : i \neq j \tag{29}$$

$$\theta_{i,b} \in \{0,1\} \qquad \forall i \in \mathbb{I}, \forall b \in \mathbb{B}_i \tag{30}$$

$$\eta_{i,b,j,b'}, \eta^{'}_{i,b,j,b'}, \eta^{''}_{i,b,j,b'} \in \{0,1\} \qquad \forall i,j \in \mathbb{I}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b^{'}\right) \vee \left(i \neq j\right)\right) \tag{31}$$

$$x_{i,b}, y_{i,b}, z_{i,b} \geq 0 \qquad \forall i \in \mathbb{I}, \forall b \in \mathbb{B}_i \tag{32}$$

$$q_i \geq 0 \qquad \forall i \in \mathbb{I}_0 \tag{33}$$

Constraint set (2) ensures that each customer is assigned to exactly one vehicle. Constraint set (3) indicates that if vehicle v is not utilized, no customer is assigned to it. Constraint sets (4)-(6) guarantee that the depot is visited at most once, and if a vehicle arrives at a customer, it must also departs from it. Constraint set (7) ensures that either customer j is not served by vehicle v or he/she is visited by v exactly once. Constraint set (8) implies that the total weight of boxes placed in a container cannot exceed its capacity. Constraint set (9) is a linear restatement of the following condition and determines the time at which each customer is visited.

$$\sum_{v\in\mathbb{V}} \delta_{i,j,v} = 1 \implies q_j \geq q_i + t_{i,j}$$

Also, this constraint prevents the formation of sub-tours. Constraint sets (10)-(14) are the linear restatements of the following conditional statements and ensure that the variable $\gamma_{i,j}$ takes correct value.

$$\begin{aligned}&\delta_{i,j,v} = 1 \implies \gamma_{i,j} = 1 \\ &\left(\gamma_{i,j} = 1 \quad \wedge \quad \gamma_{j,k} = 1\right) \implies \gamma_{i,k} = 1 \\ &\left(\beta_{i,v} = 1 \quad \wedge \quad \beta_{j,v'} = 1\right) \implies \gamma_{i,j} + \gamma_{j,i} = 0\end{aligned}$$

$$(\beta_{i,v} = 1 \quad \wedge \quad \beta_{j,v} = 1) \implies \gamma_{i,j} + \gamma_{j,i} = 1$$

Constraint sets (15)-(17) determine the position of boxes demanded by customers assigning to the same vehicle relative to each other. Indeed, constraint set (15) states that if customers i and j do not belong to the same vehicle, there is no relationship between the location of boxes of customer i and those of customer j. Constraint set (16) is a linear restatement of the following condition and prevents the overlapping of the boxes of every pair of customers assigned to the same vehicle. Also this constraint establishes the LIFO policy and guarantees that if customers i and j belong to the same vehicle and customer i is visited earlier than customer j, every box of customer j is either to the right, or to the left, or back, or below of any box of customer i. Indeed, every box of customer j cannot be an obstacle in the way of unloading the boxes of customer i.

$$\gamma_{i,j} = 1 \implies \left(\eta_{j,b',i,b} = 1 \ \vee \ \eta'_{i,b,j,b'} = 1 \ \vee \ \eta'_{j,b',i,b} = 1 \ \vee \ \eta''_{j,b',i,b} = 1\right)$$

Constraint set (17) determines the location of boxes belonging to a given customer relative to each other and prevents them from overlapping. Constraint sets (18) and (19) ensure that a non-fragile box cannot be placed on a fragile one. Constraint set (18) expresses fragility constraint for each pair of boxes belonging to two different customers, while constraint set (19) states the same issue for each pair of boxes belonging to the same customer. Constraint sets (20)-(22) determine the location of the reference point of each box where M_1, M_2 and M_3 can be set at L, W, and H, respectively. Constraint sets (23)-(25) guarantee that if a customer is assigned to a given vehicle, all of his/her boxes should be totally placed inside the container of that vehicle. Constraint sets (26)-(33) determine the types of variables.

The first part of the objective function (1) minimizes the total distance traveled by the fleet. However, the second part minimizes the leftover space between boxes and forces them to be stacked as tightly as possible where the value of parameters ρ_1, ρ_2 and ρ_3 are selected small enough so that the priority is given to minimizing the first part of the objective function; and then, among all solutions having the minimum traveled distance, the one with tighter placement of boxes is selected. More detail is provided in Remark 2.1.

Remark 2.1. *In determining the value of parameters ρ_1, ρ_2 and ρ_3, considering the fact that tight packing along the vertical axis is prior to tight packing along other axes (since boxes cannot be suspended in the container space without any supporting area), we choose the weights such that $\rho_3 > \rho_1$ and $\rho_3 > \rho_2$. Further, between two other axes (i.e., length and width), we have given the next priority to the length axis, i.e. $\rho_2 > \rho_1$. Also, since distance minimization has more priority to the tight packing, the value of ρ_1, ρ_2 and ρ_3 should be determined in such a way that for every feasible solution, the second part of the objective function ($\sum_{i\in\mathbb{I}}\sum_{b\in\mathbb{B}_i}(\rho_1 x_{i,b} + \rho_2 y_{i,b} + \rho_3 z_{i,b})$) be smaller than its first part ($\sum_{v\in\mathbb{V}}\sum_{i\in\mathbb{I}_0}\sum_{j\in\mathbb{I}_0} d_{i,j}\delta_{i,j,v}$). For this purpose, considering l_0 as the smallest distance between each pair of points in the set $\mathbb{I}_0$, the values of parameters ρ_1, ρ_2 and ρ_3 are chosen so that for every feasible solution we have:*

$$\sum_{i\in\mathbb{I}}\sum_{b\in\mathbb{B}_i}(\rho_1 x_{i,b} + \rho_2 y_{i,b} + \rho_3 z_{i,b}) < l_0$$

and since $\rho_3 > \rho_1$ and $\rho_3 > \rho_2$, it is sufficient to choose the value of ρ_3 so that:

$$\sum_{i\in\mathbb{I}}\sum_{b\in\mathbb{B}_i}(\rho_3 x_{i,b} + \rho_3 y_{i,b} + \rho_3 z_{i,b}) < l_0$$

or, equivalently,

$$\rho_3 < \frac{l_0}{\sum_{i\in\mathbb{I}}\sum_{b\in\mathbb{B}_i}(x_{i,b} + y_{i,b} + z_{i,b})}$$

Now, since $\dfrac{l_0}{\sum_{i\in\mathbb{I}}|\mathbb{B}_i|(L+H+W)} < \dfrac{l_0}{\sum_{i\in\mathbb{I}}\sum_{b\in\mathbb{B}_i}(x_{i,b} + y_{i,b} + z_{i,b})}$*, it is enough to set the value of ρ_3 as follows:*

$$\rho_3 = \frac{l_0}{\left(\sum_{i\in\mathbb{I}}|\mathbb{B}_i|(L+H+W)\right)}$$

2.3. Valid inequalities and symmetry-breaking constraints

Although the volume-capacity is satisfied by constraints (23)-(25), the following inequality is a valid cut indicating the observation of the volume-capacity:

$$\sum_{i\in\mathbb{I}}\beta_{i,v}\sum_{b\in\mathbb{B}_i}(l_{i,b}\times w_{i,b}\times h_{i,b}) \le (L\times W\times H)\,\alpha_v \qquad \forall v\in\mathbb{V} \tag{34}$$

Also, since the vehicles are assumed to be homogeneous, the proposed model has symmetric solutions. We illustrate this concept by a simple example. Suppose that the demands of five customers should be delivered by two homogeneous vehicles (i.e., n=5,V=2). Consider the solution in which the routes associated with vehicles 1 and 2 are $0 \to 1 \to 2 \to 3 \to 0$, and $0 \to 4 \to 5 \to 0$, respectively. Due to the homogeneity of vehicles, this solution is similar to the one obtained by swapping the aforementioned routes between vehicles, i.e., the solution in which vehicles 1 and 2 travels on the routes $0 \to 4 \to 5 \to 0$ and $0 \to 1 \to 2 \to 3 \to 0$, correspondingly. We refer to these equivalent solutions as symmetric solutions. In the general case with V homogeneous vehicles, in any solution with R specific routes, the routes can be assigned to vehicles in $R!$ equivalent ways. Thus, symmetric solutions enlarge the solution space and cause exact methods such as branch-and-bound become ineffective in the resolution of the problem. To eliminate the symmetry, we use a rule based on a lexicographic order on the set of customers. According to this rule, the route containing customer 1 is always assigned to vehicle 1; then, vehicle 2 travels on the route having the smallest customer among those not allocated to vehicle 1. Similarly, vehicle 3 travels on the route having the smallest customer among those not assigned to vehicles 1 and 2, and the same process continues. To incorporate this rule into the model, the symmetry-breaking constraints (35)-(37) are introduced. For more details, see [24].

$$\alpha_{v+1} \leq \alpha_v \qquad \forall v \in \mathbb{V} :< |\mathbb{V}| \tag{35}$$

$$\sum_{\substack{v\in\mathbb{V}\\ v\leq j}} \sum_{i\in\mathbb{I}_0} \delta_{i,j,v} = 1 \qquad \forall j \in \mathbb{I} : j \leq \min\{|\mathbb{I}|, |\mathbb{V}|\} \tag{36}$$

$$\sum_{v'=v}^{\min\{j,v\}} \sum_{\substack{i\in\mathbb{I}_0\\ i\neq j}} \delta_{i,j,v'} \leq \sum_{\substack{j'\in\mathbb{I}\\ v-1\leq j'\leq j-1}} \sum_{\substack{i\in\mathbb{I}_0\\ i\neq j'}} \delta_{i,j',v-1} \qquad \forall j \in \mathbb{I}, \forall v \in \mathbb{V} \ : j \geq v, v > 1 \tag{37}$$

Constraint set (35) ensures that the vehicles are used in order; in other words, while vehicle v is not used, vehicles $v^{'}$ with $v^{'} > v$ are not allowed to be utilized. Constraint set (36) implies that customer 1 is assigned to vehicle 1, and in general, each customer j is assigned to one of vehicles v with $v \leq j$. Constraint set (37) guarantees that for every vehicle v and each customer j, if none of the customers $j^{'}$ with $j^{'} < j$ are assigned to vehicle $v-1$, customer j can not be assigned to vehicle $v^{'}$ with $v^{'} \geq v$.

Throughout the rest of this paper, we refer to valid inequality 34 and the symmetry-breaking constraints (35)-(37) as valid cuts, for short.

3. Evaluation of the proposed model

First, we consider a small numerical instance, taken from [12], including 15 customers and four vehicles where the total number of boxes demanded by different customers equals 26. The length, width, height, and weight-capacity of the container of each vehicle are equal to 60cm, 25cm, 30cm, and 90kg, respectively. Further, the distance between points and the characteristics of boxes demanded by each customer are given in Table 1 and Table 2, correspondingly. The model associated with this instance includes 4256 binary variables, 112 continuous variables and 12312 constraints (including the valid cuts (34)-(37). The model is implemented in GAMS software and solved via the solver CPLEX. The optimal value of the total traveled distance equals 297.7, and the optimal routes are as follows:

The route corresponding to vehicle 1: $0 \to 2 \to 3 \to 8 \to 1 \to 0$

The route corresponding to vehicle 2: $0 \to 4 \to 15 \to 10 \to 9 \to 5 \to 12 \to 0$

The route corresponding to vehicle 3: $0 \to 6 \to 7 \to 14 \to 13 \to 0$

The route corresponding to vehicle 4: $0 \to 11 \to 0$

Figure 3 through Figure 5 show how the boxes demanded by the customers of each route are placed in the container. In these figures, on each box, a code containing two numbers, separated by a hyphen, is put. The left number shows the customer demanding that box, and the right one represents the box number. For example, the code 1-3 refers to the third box demanded by customer 1. Further, boxes containing fragile goods are marked with a white cross. To show how boxes are stacked upon each other in the container, different layers from bottom to top are separately depicted. Layers are numbered sequentially by starting at $L = 1$ for the most bottom layer.

Table 3 shows the results of solving the proposed model via CPLEX on other small-sized instances, taken from [12] within a time-limit of 7200 seconds. The first four columns of this table show the problem characteristics. Columns labeled by No.BVar, No.CVar, and No.Const represent the number of binary and continuous variables and the number of constraints, respectively. The column TTD shows the total traveled distance associated with the best solution obtained by CPLEX within a time-limit of 7200 seconds, and the last column represents the relative

Table 1: Distance between points

j i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0	-	13.9	21.0	32.6	17.2	14.1	11.4	26.4	22.0	23.1	28.3	12.0	8.1	29.2	18.1	24.7
1	13.9	-	12.4	19.2	31.1	22.2	16.8	22.8	11.7	24.2	34.0	12.1	20.9	41.9	26.9	36.0
2	21.0	12.4	-	15.3	37.0	21.0	28.1	34.9	22.2	16.3	28.1	10.6	24.8	50.1	37.7	35.5
3	32.6	19.2	15.3	-	49.7	36.1	35.4	35.0	21.1	31.0	43.0	25.1	38.3	61.1	45.6	50.6
4	17.2	31.1	37.0	49.7	-	20.4	21.0	37.1	37.6	32.8	31.4	26.6	12.5	15.0	17.9	18.9
5	14.1	22.2	21.0	36.1	20.4	-	25.5	40.2	33.2	12.4	14.2	11.2	9.2	35.4	30.5	14.6
6	11.4	16.8	28.1	35.4	21.0	25.5	-	16.5	18.0	34.0	39.7	21.8	18.0	27.2	10.3	34.4
7	26.4	22.2	34.9	35.0	37.1	40.2	16.5	-	14.0	46.1	54.0	33.3	34.0	39.85	21.6	50.7
8	22.0	11.7	22.2	21.1	37.6	33.2	18.0	14.0	-	35.8	45.6	23.7	30.0	45.2	27.6	46.3
9	23.1	24.2	16.3	31.0	32.8	12.4	34.0	46.1	35.8	-	12.0	12.8	21.0	47.7	41.0	23.3
10	28.3	34.0	28.1	43.0	31.4	14.2	39.7	54.0	45.6	12.0	-	22.0	22.8	46.2	44.3	15.8
11	12.0	12.1	10.6	25.1	26.6	11.2	21.8	33.3	23.7	12.8	22.0	-	14.2	40.3	30.0	25.7
12	8.1	20.9	24.8	38.3	12.5	9.2	18.0	34.0	30.0	21.0	22.8	14.2	-	26.9	21.5	16.8
13	29.2	41.9	50.1	61.1	15.0	35.4	27.2	39.9	45.2	47.7	46.2	40.3	26.9	-	18.4	32.3
14	18.1	26.9	37.7	45.6	17.9	30.5	10.3	21.6	27.6	41.0	44.3	30.0	21.5	18.4	-	35.4

Table 2: Distance between points

Customer	**Box number**	**Length**	**Width**	**Height**	**Weight**	**Fragility**
1	1	30	5	7	7	1
2	1	29	8	15	30	1
3	1	33	15	16	8	0
3	2	36	5	6	8	0
4	1	15	15	17	9	0
5	1	13	7	15	10.5	0
5	2	15	10	8	10.5	0
6	1	12	14	12	5	0
6	2	27	11	6	5	0
6	3	16	9	20	5	1
7	1	23	7	10	9.5	0
7	2	21	7	10	9.5	1
8	1	15	14	12	7.66	0
8	2	27	8	7	7.66	0
8	3	31	6	9	7.66	1
9	1	24	7	10	11	0
10	1	25	12	14	5	0
11	1	31	15	15	6.33	0
11	2	19	13	14	6.33	0
11	3	16	13	10	6.33	0
12	1	31	8	9	9.66	0
12	2	21	7	14	9.66	0
12	3	29	7	7	9.66	0
13	1	34	11	16	7.66	0
13	2	26	13	17	7.66	0
13	3	28	10	11	7.66	0
14	1	27	13	14	7	0
14	2	33	11	12	7	0
14	3	17	10	8	7	0
15	1	33	12	18	3.33	0
15	2	23	12	9	3.33	1
15	3	34	6	9	3.33	1

gap in percent reported by CPLEX. The symbol "-" placed in some entries of the last two columns indicates that CPLEX is unable to find any feasible solution to the corresponding instance within the given time-limit.

Table 4 evaluates the effect of adding valid inequalities. For this purpose, the linear-programming (LP) relaxation bound and the objective value corresponding to the best solution obtained by CPLEX within a time-limit of 7200 seconds are reported in two cases "without valid cuts" and "with valid cuts". As can be seen, considering valid cuts can lead to a better LP relaxation bound and accordingly, accelerate the achievement of a high quality solution.

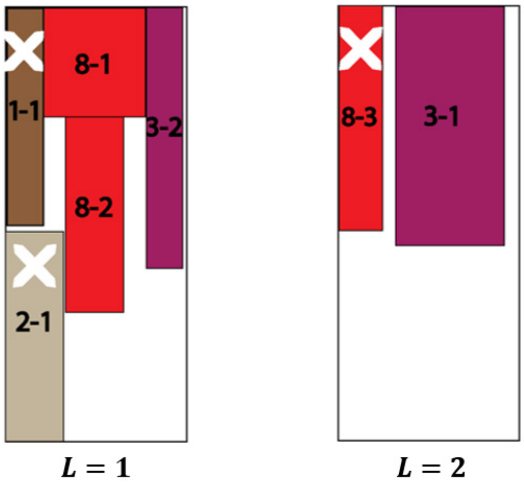

Figure 3: Boxes demanded by the customers of the route $0 \to 2 \to 3 \to 8 \to 1 \to 0$

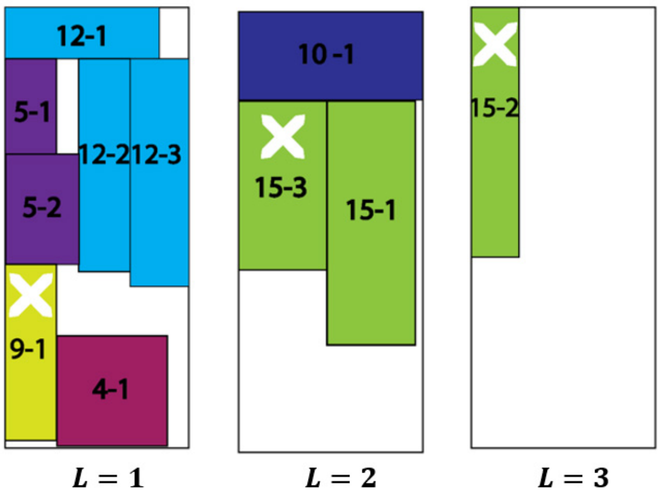

Figure 4: Boxes demanded by the customers of the route $0 \to 4 \to 15 \to 10 \to 9 \to 5 \to 12 \to 0$

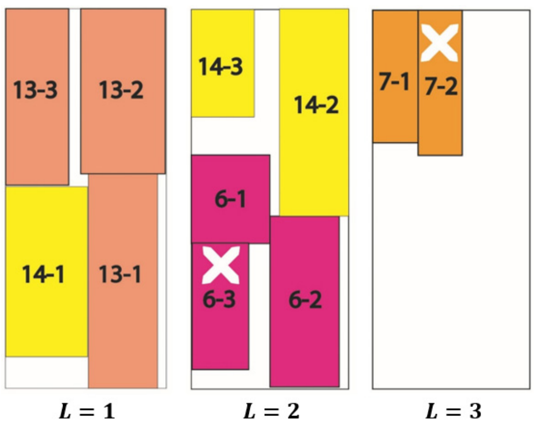

Figure 5: Boxes demanded by the customers of the route $0 \to 6 \to 7 \to 14 \to 13 \to 0$

4. Model-based packing procedure in meta-heuristics

Due to the NP-hardness of the problem 3VRLP, exact methods are unable to optimally solve its large-sized instances. Thus, meta-heuristic algorithms have been used in the literature to produce near optimal solutions within a reasonable time. In most of the meta-heuristic algorithms, presented for 3VRLP, a two-stage framework is followed so that the meta-heuristic, as the main body, controls the neighborhood exploration for routing decisions, while

Table 3: Results of the proposed model on small-sized instances taken from [12]

ID	$\lvert\mathbb{I}\rvert$	$\sum_{i\in\mathbb{B}_i}\lvert\mathbb{B}_i\rvert$	$\lvert\mathbb{V}\rvert$	No.BVar	No.CVar	No.Const	TTD	Gap (%)
1	15	32	4	4266	112	12312	297.7	0
2	15	26	5	3466	94	12914	335.0	0
3	20	37	4	6176	132	25709	378.4	29.2
4	20	36	6	6841	129	30021	453.2	15.4
5	21	45	6	9309	157	36380	947.7	66.1
6	21	40	6	8044	142	34413	510.5	22.9
7	22	46	6	9891	161	39933	-	-

Table 4: Evaluating the performance of valid cuts

ID	$\lvert\mathbb{I}\rvert$	$\sum_{i\in\mathbb{B}_i}\lvert\mathbb{B}_i\rvert$	$\lvert\mathbb{V}\rvert$	LP relaxation bound		Gap reported by CPLEX (%)		Time (s)	
				With valid Cuts	Without Valid cuts	With valid Cuts	Without Valid cuts	With valid Cuts	Without Valid cuts
1	15	32	4	230.1	230.1	0	0.01	3866	>7200
2	15	26	5	264.7	264.7	0	0.01	7100	>7200
3	20	37	4	273.8	273.8	29.2	33.5	>7200	>7200
4	20	36	6	325.4	325.4	15.4	23.8	>7200	>7200
5	21	45	6	516.4	503.9	66.1	-	>7200	>7200
6	21	40	6	318.8	301.6	22.9	28.1	>7200	>7200
7	22	46	6	457	418.5	-	-	>7200	>7200

some packing procedures are called iteratively to check the feasibility of routes generated during the meta-heuristic with respect to the loading constraints (e.g. not-overlapping, fragility, rotation, and LIFO). If the packing procedure cannot find a feasible loading for a given solution obtained by neighborhood exploration, that solution is ignored; otherwise, its fitness is calculated. In this regard, Gendreau et al. [12] and Zhu et al. [33] used TS as the main body and proposed two greedy heuristics, namely touching area and bottom-left-fill as packing procedures to check loading feasibility. Tarantilis et al. [30] applied TS and presented a collection of packing heuristics for loading part. Fuellerer et al. [11] and Ruan et al. [28] utilized the ant-colony algorithm and honey-bee optimization, respectively, and applied the same heuristics as Gendreau et al. [12] for loading. Bortfeld [4] used TS as the main body and the tree-search algorithm as the loading procedure. Tao and Wang [29] applied TS as the main body and proposed an improved least-waste heuristic for loading part. Krebs and Ehmke [18] included axle weight constraint into the 3VRLP and solved it by a hybrid heuristic approach containing the outer adaptive large neighborhood search as the main body and the bottom-left-fill as the packing procedure. As can be seen, touching area, bottom-left-fill, and the least-waste algorithms, are amongst the most used loading procedures. However, in cases that the limited space of the container should contain a lot of number of boxes very compactly, loading procedures may become unable to reach a feasible placement. That is, although there may be a feasible loading, such procedures fail to find it. Thus, the corresponding solution is ignored as an infeasible solution while it is feasible. To overcome this weakness, in this section, we introduce a loading MILP, as a restricted version of the model 3VRLP, which can be efficiently used for feasibility checking in combination with existing loading procedures.

4.1. MILP model for loading

Let $\mathbb{I}'$ be the set of customers visited in the route of a given vehicle, and consider O_i as the order of visiting customer $i \in \mathbb{I}'$ in this route. The aim is to find a feasible placement of boxes demanded by the customers of this route in the container so that the constraints of rotation in horizontal plane, non-overlapping, fragility and LIFO are satisfied. To this end, we present a loading-check MILP model, extracted from the model 3VRLP, and refer to it as LCP. The infeasibility status of this model means that there is no feasible placement of boxes for the given route. The variables of this model have the same definition as the model 3VRLP with the difference that instead of $\mathbb{I}$, they are stated over the set $\mathbb{I}'$. LCP is formulated as follows:
(LCP)

$$\min \sum_{i\in\mathbb{I}'}\sum_{b\in\mathbb{B}_i}(\rho_1 x_{i,b} + \rho_2 y_{i,b} + \rho_3 z_{i,b}) \tag{38}$$

s.t.

$$\eta_{j,b',i,b} + \eta^{'}_{i,b,j,b'} + \eta^{'}_{j,b',i,b} + \eta^{''}_{j,b',i,b} \geq 1 \qquad \forall i,j \in \mathbb{I}^{'} : i \neq j, O_i < O_j, \forall b \in \mathbb{B}_i,\ \forall b^{'} \in \mathbb{B}_j \tag{39}$$

$$\eta_{i,b,i,b'} + \eta_{i,b',i,b} + \eta^{'}_{i,b,i,b'} + \eta^{'}_{i,b',i,b} + \eta^{''}_{i,b,i,b'} + \eta^{''}_{i,b',i,b} \geq 1 \qquad \forall i \in \mathbb{I}^{'}, \forall b, b^{'} \in \mathbb{B}_i : b < b^{'} \tag{40}$$

$$\eta_{i,b,j,b'} + \eta_{j,b',i,b} + \eta^{'}_{i,b,j,b'} + \eta^{'}_{j,b',i,b} + \eta^{''}_{j,b',i,b} \geq 1 \qquad \forall i,j \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : i \neq j, f_{i,b} = 1,\ f_{j,b'} = 0 \tag{41}$$

$$\eta_{i,b,i,b'} + \eta_{i,b',i,b} + \eta^{'}_{i,b,i,b'} + \eta^{'}_{i,b',i,b} + \eta^{''}_{i,b',i,b} \geq 1 \qquad \forall i \in \mathbb{I}^{'}, \forall b, b^{'} \in \mathbb{B}_i : b \neq b^{'},\ f_{i,b} = 1,\ f_{i,b'} = 0 \tag{42}$$

$$x_{i,b} + l_{i,b}\theta_{i,b} + w_{i,b}\left(1 - \theta_{i,b}\right) \leq x_{j,b'} + M_1\left(1 - \eta_{i,b,j,b'}\right)$$
$$\forall i,j \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b^{'}\right) \vee (i \neq j)\right) \tag{43}$$

$$y_{i,b} + w_{i,b}\theta_{i,b} + l_{i,b}\left(1 - \theta_{i,b}\right) \leq y_{j,b'} + M_2\left(1 - \eta^{'}_{i,b,j,b'}\right)$$
$$\forall i,j \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b^{'}\right) \vee (i \neq j)\right) \tag{44}$$

$$z_{i,b} + h_{i,b} \leq z_{j,b'} + M_3\left(1 - \eta^{''}_{i,b,j,b'}\right) \qquad \forall i,j \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b^{'}\right) \vee (i \neq j)\right) \tag{45}$$

$$x_{i,b} + l_{i,b}\theta_{i,b} + w_{i,b}\left(1 - \theta_{i,b}\right) \leq L \qquad \forall i \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i \tag{46}$$

$$y_{i,b} + w_{i,b}\theta_{i,b} + l_{i,b}\left(1 - \theta_{i,b}\right) \leq W \qquad \forall i \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i \tag{47}$$

$$z_{i,b} + h_{i,b} \leq H \qquad \forall i \in \mathbb{I}^{'} \forall b \in \mathbb{B}_i \tag{48}$$

$$\theta_{i,b} \in \{0,1\} \qquad \forall i \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i \tag{49}$$

$$\eta_{i,b,j,b'}, \eta^{'}_{i,b,j,b'}, \eta^{''}_{i,b,j,b'} \in \{0,1\} \qquad \forall i,j \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i, b^{'} \in \mathbb{B}_j : \left(\left(i = j\ \wedge b \neq b^{'}\right) \vee (i \neq j)\right) \tag{50}$$

$$x_{i,b}, y_{i,b}, z_{i,b} \geq 0 \qquad \forall i \in \mathbb{I}^{'}, \forall b \in \mathbb{B}_i \tag{51}$$

Objective function (38) seeks a feasible loading with the lowest empty space between boxes. Constraint set (39) prevents the overlapping of the boxes of every pair of customers $i, j \in \mathbb{I}^{'}$. Also, this constraint establishes the LIFO policy. Constraint set (40) prevents the boxes of the same customer from overlapping. Constraint sets (41) and (42) are restatements of constraint sets (18) and (19) for $i, j \in \mathbb{I}^{'}$. Constraint sets (43)-(48) have the same description as (20)-(25) with the difference that they are stated over the set $\mathbb{I}^{'}$. Constraint sets (49)-(51) determine the types of variables.

4.2. Incorporating the model LCP into the TS algorithm of Tao and Wang [29]

Since the assumptions of the problem addressed in this paper is similar to the one studied by Tao and Wang [29], here, we consider the TS, proposed by Tao and Wang [29], as a base, and explain how their loading algorithm can be combined with the model LCP. For the sake of briefness, we will not repeat the details of their algorithm and just focus on the differences.

The TS proposed by Tao and Wang [29] seeks the search space by implementing some operators including customer swapping within one route, transferring a customer to another route, and crossover, and once any new route is generated, the least-waste packing procedure is called. If no feasible loading is obtained by this procedure, the corresponding solution is ignored, and another neighbor is examined; otherwise, that solution is accepted as a valid neighbor. We change this part of their algorithm in such a way that in the cases the packing procedure fails to find a feasible loading, the model LCP is solved via a MILP solver. Of course, a stopping condition is set on the solution process of LCP so that as soon as the first feasible solution is found or the infeasibility of the model is proved, it is terminated. If the LCP status is infeasible, the corresponding solution is ignored, and another neighbor is examined; otherwise, that solution is accepted as a valid neighbor.

The modified algorithm was conducted on instances taken from Tao and Wang [29] on a laptop running Windows 10 operating system with a Core(TM) i5 processor, and 8.0 GB of RAM. TS was coded in Python and the optimization models were solved by CPLEX solver, included in the GAMS software by utilizing the GAMS-Python API. The results indicated that the modified algorithm can provide an averaged improvement of 0.85% compared to the TS of Tao and Wang [29]. The running time of the modified algorithm was about 3146 seconds, on average which is about 1.2 times the running time of the TS algorithm of Tao and Wang [29]. This time difference is due to the resolution of MILP model LCP in some implementations of the packing phase. With respect to the performance of the proposed algorithm, the incorporation of the model LCP into the meta-heuristic algorithms of 3VRLP would be valuable.

4.3. Conclusions

In this paper, a novel MILP model together with some valid cuts was presented for vehicle routing and loading problem with fragility, rotation, and LIFO constraints. Further, we introduced the model LCP and showed that the performance of the loading procedures in the meta-heuristics can be improved by combining them with LCP. The extension of the proposed model to consider other constraints such as simultaneous pick-up and delivery and appropriate supporting area is suggested as a future work.

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