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Review Article

Vehicle scheduling problem under uncertainty: literature review and future perspective

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ABSTRACT: Vehicle scheduling problem is an important combinatorial optimization problem arising in the management of transportation companies. The problem consists of assigning a set of timetabled trips to a set of vehicles to minimize a given objective function. In this paper, vehicle scheduling problem under undeterministic conditions is considered. The paper states the necessity of considering the uncertain condition in the problem and reviews the different solution approaches to deal with the conditions of uncertainty. The main objective of this paper is to discuss the importance of considering uncertainty in vehicle scheduling problem, review the relevant literature and present some directions for future work.

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(Dedicated to Professor S. Mehdi Tashakkori Hashemi)

1. Introduction

Vehicle scheduling is one of the most important problems in the transportation system planning process, which aims to provide a suitable scheduling for the existing vehicles to meet the system requirements and to perform trips optimally ([6], [41]). In transportation networks, the vehicle's schedule is based on a pre-designed timetable and remains unchanged until the timetable changes [16]. During the implementation of this schedule, due to the uncertain nature of transportation networks (occurrence of unpredictable events, traffic conditions, disruption of the vehicle, weather conditions, equipment failure and variable traffic light schedule) a number of trips may be delayed. If there is a delay, this delay will cause more delays.

In some researches, to avoid delays in the vehicle schedules, a fixed storage time was added to the transfer time [9]. Although increasing storage time can reduce latency to some extent, it increases the cost of scheduling. In fact, in many cases there is no need to add storage time. Furthermore, there is no way to determine the storage time. As an example, consider the following conditions [61]:

Consider the vehicle scheduling problem with the aim of minimizing the number of vehicles. The goal is to provide an optimal schedule for each vehicle in one depot to perform a set of four trips. Table 1 shows the start time, end time, start station and end station of these trips.

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Table 1: An example of vehicle scheduling problem.

Trip	Strat time	End time	Strat station	End station
1	09:00	10:00	B	A
2	09:15	10:00	C	A
3	10:05	11:05	A	B
4	10:15	11:00	A	C

An optimal solution to this problem is to allocate trips 1 and 3 to vehicle 1 and trips 2 and 4 to vehicle 2. Suppose a ten-minute delay in trip 1 occurs. In this case, if we use the same schedule as before, the trip 3 will start with a five-minute delay. Even if a storage time of four minutes (or less) is considered for each trip, trip 3 will still start with a delay.

A rescheduling approach is also proposed to avoid trip delays [8]. If in the example above we know that trip 1 is delayed and trip 2 is done without delay, we can change the schedule so that trip 3 is done without delay. However, this method also has disadvantages that make it impossible to do in real conditions. In fact, the implementation of this method requires accurate knowledge of the delay, which is determined only at the time of travel [67]. Therefore, a communication and information system is required.

Another proposed approach was stochastic programming to provide sustainable scheduling for vehicles ([15],[40]). Although, the proposed approach has been able to reduce the latency and expected cost of scheduling compared to the method of adding storage time, but in practice, when considering real networks, the complexity of the problem is very high and the approach is only able to provide scheduling in small and medium-sized networks ([25],[18]). Therefore, it is not efficient for large-scale transportation networks.

The following explains the reasons for the uncertainty in the vehicle scheduling problem.

- The capacity of the route is one of the influential parameters during the trip, which depends on external events such as accidents, weather conditions, the number of vehicles stopped along the route, etc., which are unpredictable.
- Traffic conditions on the routes are monitored by image processing cameras and speed cameras, which are regulated by human operators (human agents). This causes different and unpredictable estimates of traffic conditions in different conditions, which will cause uncertainty during the trip.
- The variability of the queue length at the intersections with traffic lights will cause the variable scheduling of the traffic lights and uncertainty during the trip.
- Intelligent information systems in dealing with route guidance activities in such a way as to distribute traffic across the network. If drivers do not pay attention to the route offer, they may cause traffic imbalance in the network. Depending on the level of trust of drivers, we have different levels of traffic in the network, which due to the unpredictability of user behavior, the level of network traffic and therefore travel time will be unpredictable. Also, users usually do not have accurate information about the level of network traffic.
- Uncertainty in travel time, driver schedule and schedule of maintenance activities, causes uncertainty in the departure time of the vehicles.

Since we face with linguistic and qualitative descriptions in vehicle scheduling problem, and the stochastic approaches do not represent a good representation of the real environment [11], another useful approach is fuzzy approach to deal with uncertainty conditions [20]. For example, when the station is crowded, the instruction can be to increase the services, which is a qualitative description, or in calculating the travel time parameter, we are faced with linguistic variables such as heavy, semi-heavy and light traffic [50]. In addition, in fuzzy models, it is possible to describe human behavior such as risk-taking behavior of system administrators.

On the other hand, calculations in discrete probabilistic space are very complex and somewhat impractical [56]. Therefore, this solution cannot always meet the requirements of the problem for modeling real conditions and solving large-scale instances in conditions of uncertainty. In addition, in probabilistic models, it is often difficult to find the probabilistic distribution of a parameter ([68],[69]). Therefore, fuzzy approach provided with more adaptability to the real conditions in the vehicle scheduling problem [48].

In addition, fuzzy optimization models can usually be converted to integer programming models and can be solved with all optimization tools, including classical, heuristic, and meta-heuristic algorithms. Note that factors influencing vehicle scheduling include vehicle cost, transfer cost, and travel time dependent on unpredictable events,

variable traffic light scheduling, fleet cost, lane scheduling, driver scheduling and demand patterns which are generally indecisive and unpredictable [52]. Therefore, in the vehicle scheduling problem, the effect of the uncertainty of the above factors must be carefully considered.
The paper is organized as follows: the sources of uncertainty in the vehicle scheduling problem are explained in Section 2. In addition, in this section, different approaches to solve the vehicle scheduling problem are reviewed and the advantages and disadvantages of each method are mentioned. At the end of this section, the latest papers on vehicle scheduling problem under uncertainty are collected. In Section 3, the research gap is presented and future perspectives in this field are proposed. Finally, the paper ends with a brief conclusion and future directions in Section 4.

2. Literature Review

The transportation planning process consists of the following six steps: Transport Network Design Problem(TNDP), Frequency Scheduling Problem (FSP), Time-tabling Problem (TP), Vehicle Scheduling Problem (VSP), Driver Scheduling Problem (DSP), and Fleet Maintenance Scheduling Problems(FMSP) [13]. Usually in scientific researches, transportation planning problems are examined in two parts: design, scheduling and their combination([28],[59]). Figure 1 shows the general classification of these problems and their relationship to each other.

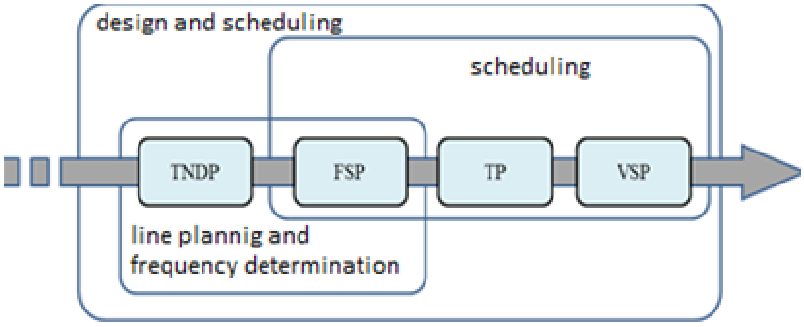

Figure 1: Transportation system management process and classification of the related problems [22].

The vehicle scheduling problem has been extensively investigated in the literature [41]. Due to the NP-hardness of the problem, polynomial time algorithms cannot be used to solve the problem in real conditions [29]. Various ideas for solving this problem include a wide range of methods such as decomposition methods, integer programming approaches, local search methods, and meta-heuristic algorithms[36]. Decomposition methods such as the column generation method and the Lagrangian relaxation method produce relatively accurate solutions at high computational times. On the other hand, meta-heuristic algorithms such as tabu search methods produce a suitable answer in a short computational time without guaranteeing optimality, which is significant in large-scale problems.
Commonly, heuristic and meta-heuristic methods have been considered by researchers([57],[71]). These algorithms generate schedules in low computational time, but the accuracy of the generated solution is low [4]. Since, the generated schedules is costly to implement, a number of researchers emphasize that it is cost-effective to provide an accurate model with reasonable complexity to generate a more accurate schedule [32]. According to this view, heuristic and meta-heuristic algorithms are only suitable for problems that require quick answers (such as the routing problem). However, a group of researchers emphasize the use of heuristic methods due to the ability of these methods to solve the problem quickly and the possibility of improving the answers obtained [19]. They also emphasize the flexibility of heuristic methods to consider the constraints and goals that need to be added to the model to manage the system. Compared to other algorithms, heuristic algorithms can be combined with other methods and it is possible to improve the results of the algorithm by combining it with other algorithms. In addition, in most heuristic and meta-heuristic algorithms, parallel processing can be used to accelerate the algorithm. However, since heuristic algorithms are based on the specific characteristics of the problem, the stability of the algorithm for solving similar problems is low and requires appropriate modifications.
Accurate methods, including integer programming methods, allow to generate more accurate schedules [60]. Although, it is almost impossible to solve this problem in real-time networks, it has been suggested by a number of researchers because of its ability to generate accurate schedules. Of course, it is emphasized that as the size of the problem increases, these methods are not able to solve the problem [33].
Decomposition methods to decompose the problem into subproblems and integrating the subproblems into a whole are highly efficient [7]. Decomposition algorithms including Lagrangian relaxation and Dantzig-Wolf decomposition

Table 2: Investigation and comparison of the algorithms proposed for vehicle scheduling problems

Algorithm	Decomposition methods	Integer programming approaches	Heuristic and meta-heuristic algorithms
Accuracy	***	***	*
Time	**	*	***
Ability to solve real and large scale problems	***	*	***
Simplicity and flexibility of the method	**	*	***
Ability to parallelize	***	*	**
Ability to combine with other algorithms	***	*	***
Stability	***	***	*

have been emphasized for this problem due to the large-scale of the problem([34],[46]). These algorithms are capable of producing a relatively accurate answer with a low optimality gap at the appropriate execution time [43]. In these methods it is possible to perform parallel processing to accelerate the production of the solution [47]. On the other hand, the stability of these methods is such that the proposed method can be used in similar problems and transportation networks with minor changes [17].
The qualitative results of comparing these algorithms are summarized in Table 2. In this table, the superiority of one algorithm over other algorithms is indicated by "*" symbols.

Also, Table 1 summarizes the recent researches carried out on the vehicle scheduling problems under uncertainty.

3. Research Gap

Over the last few decades, network flow problems in fuzzy conditions have attracted a great deal of attention [39]. Although the vehicle scheduling problem has been studied from different perspectives, due to the complexity of problem, the proposed solutions are usually based on the assumption that the parameters are crisp and also the real situation of the problem is ignored ([37],[65]). Few studies have been conducted on vehicle scheduling with time-varying travel time, which due to the rapid increase in the dimension of the problem, these methods are not feasible in large-scale networks [27]. For these reasons, providing a solution approach that can address the uncertainty conditions in the vehicle scheduling and also provide the ability to model the real situation of the problem, can be effective in improving the management methods of the transportation systems. However, no significant studies have been performed to address this issue.
Also, considering the existence of several depots in the vehicle scheduling problem [24], it is necessary to investigate the multi-commodity flow problem in fuzzy conditions and in general to investigate mathematical programming problems with block angular structure in fuzzy conditions.
Due to the fact that decomposition algorithms have the ability to decompose the problems into subproblems and to integrate the subproblems solution approaches into a general scheme, Danzig-Wolf decomposition method can be used to solve fuzzy block angular structure problems, especially multi-commodity flow problems in fuzzy conditions. Then, the branch and price algorithms can be proposed to generate the correct answer. Although, fuzzy mathematical programming problems have been widely considered in the literature ([22], [58], [10], [12], [38], [14], [53], [35], [54], [42], [45]), researches on block angular structure problems in fuzzy conditions is scarce. The results can be used to solve multi-commodity flow problems in fuzzy conditions. In this problem, cost, capacity and demand parameters are formulated as fuzzy parameters. So, in this case we are dealing with a fuzzy objective function and fuzzy constraints [63]. Since the multi-depot vehicle scheduling problem is NP-hard [3] and we are dealing with large-scale networks in real conditions, the combined solution approaches based on the combination of branch and price and heuristic algorithms are proposed ([1],[5],[52]). In addition, a neural network and heuristic algorithms can be used to estimate the problem parameters.

4. Conclusion

This paper provides an overview of existing methods for solving the vehicle scheduling problem under uncertainty. The problem is also investigated in real applications and a research gap is presented. In this paper, the uncertainty

Table 3: A summary of researches carried out on the vehicle scheduling problem under uncertainty.

Reference	Year	Uncertainty consideration	Proposed approach
Ucar, et al. [41]	2017	Managing disruptions	Exact simultaneous column-and-row generation algorithm
Wang, et al. [63]	2020	Traffic congestion	Multi-objective genetic algorithm, departure time adjustment procedure
Filabadi et al. [15]	2022	Stochastic arrival times, alighting fraction, demands, running time of vehicles	Approximation algorithm
Yu and Shen [69]	2021	Random trip time, nonnegligible service time, possible customer cancellations	K-means clustering-based algorithms
He, et al. [26]	2018	Stochastic trip times	Approximate dynamic programming approach
Shen et al. [56]	2016	Variability of traffic and driving conditions	Hybrid heuristic
Naumann et al. [39]	2011	Managing disruptions	Stochastic programming
Lieshout et al. [62]	2018	Different scenarios for future travel times	Iterative neighborhood exploration heuristic
Shahparvari and Abbasi [55]	2011	evacuee population, time windows and bushfire propagation	Stochastic programming
Yan and Tang [68]	2009	Variable market shares and uncertain market demands	Stochastic and robust programming
Wei [65]	2015	Fuzzy travel times	Chance-constrained programming
Dávid and Krész [8]	2017	Managing disruption	Heuristic algorithms
Wagale et al. [66]	2013	Bus traffic costs	Demand- and Travel time Responsive (DTR) model
Mehrabian et al. [37]	2017	demand, due dates and processing times	Non-dominated Sorting Genetic Algorithm (NSGAII) and multi-objective particle swarm optimization (MOPSO)

conditions in the problem are emphasized according to the real applications and various sources of uncertainty. Also, various solution approaches to deal with uncertainty situations have been reviewed and compared. The paper mainly focuses to discuss the importance of considering uncertainty in vehicle scheduling problem, review the relevant literature and present some directions for future work.

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