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Locally projectively flatness and locally dually flatness of generalized Kropina conformal change of m-th root metric

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ABSTRACT: In this paper, we consider the generalized Kropina conformal change of m-th root metric and for this, prove a necessary and sufficient condition of locally projectively flatness. Also we proved a necessary and sufficient condition for the generalized Kropina conformal change of m-th root metric is locally dually flat.

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1. Introduction

The conformal theory which is based on the theory of Finsler spaces given by Matsumoto [10] and has been developed by M. Hashiguchi [7]. Suppose F and $\bar{F}$ be two metrics on a manifold as $\bar{F} = e^{\sigma(x)}F$, where σ is a scalar function on manifold, then two metrics F and $\bar{F}$ are conformally related. The conformal change is said to be a homothety if σ is a constant or $\sigma_i = \frac{\partial \sigma}{\partial x^i} = 0$ and isometry if $\sigma = 0$.

In 1979, H. Shimada [15] developed the theory of m-th root metrics and was later applied to ecology by Antonelli [2]. A fourth root metric in form $F = \sqrt[4]{y^1y^2y^3y^4}$ is called Berwald-Moór metric [4], which is a crucial subject for physicists. m-th root metrics have been studied by many authors ([3], [5], [6], [9], [11], [14] and [19]). Shen and Li in [9] have studied the geometric properties of $F = \sqrt[4]{a_{ijkl}(x)y^iy^jy^ky^l}$ and $F = \sqrt{\sqrt{a_{ijkl}(x)y^iy^jy^ky^l} + b_{ij}(x)y^iy^j}$.

In this paper, we suppose generalized Kropina conformal change of m-th root metric and for this, proved a necessary and sufficient condition of locally projectively flat.
Suppose (M, F) be a Finsler space. We define a Finsler change as follows:

$$F(x, y) \to \bar{F}(x, y) = f(F, \beta),$$

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where $f(F,\beta)$ is a positively homogeneous function of F and $\beta(x,y)=b_i(x)y^i$. A Finsler change $\bar{F}=\frac{F^{k+1}}{\beta^k}$, called generalized Kropina change of metric F.

In this paper, we suppose Finsler change, called generalized Kropina conformal change of metric F, as

$$\bar{F}=e^{\sigma(x)}\frac{F^{k+1}}{\beta^k}, \tag{1.1}$$

where $F=\sqrt[m]{A}$ is m-th root metric, $A=a_{i_1i_2\ldots i_m}(x)y^{i_1}y^{i_2}\ldots y^{i_m}$ with $a_{i_1\ldots i_m}$ symmetric in every indices [15] and k is a positive real number.
We prove following Theorem:

Theorem 1.1. *Suppose $F=\sqrt[m]{A}$ be m-th root metric on subset $U\subset\mathbb{R}^n$, where U is an open and A is irreducible. Let $\bar{F}=e^{\sigma(x)}\frac{F^{k+1}}{\beta^k}$ is generalized Kropina conformal change of m-th root metric. Then $\bar{F}$ is locally projectively flat iff there exists $\theta=\theta_l(x)y^l$ such that*

$$A_{x^l}=m\left(A\theta_l+A_l\theta\right)+\frac{1}{2}A_l\sigma_0+\frac{m}{2(k+1)}A\sigma_{0l}-\frac{m}{2(k+1)}A\sigma_{x^l}+(k+1-m)\theta A_l, \tag{1.2}$$

$$\beta_{x^l}=\frac{(k+1)}{mA}A_0b_l+\frac{(k+1)^2}{m^2A}A_l\beta_0+\beta_{0l}+b_l\sigma_0 \tag{1.3}$$

and

$$\beta_0b_l=0,$$

where $A_0=A_{x^t}y^t$, $A_l=A_{y^l}$, $\beta_0=\beta_{x^t}y^t$, $\beta_{0l}=\beta_{x^ty^l}y^t$, $b_l=\beta_l=\beta_{y^l}$, $\sigma_0=\sigma_{x^t}y^t$, $\sigma_{0l}=\sigma_{x^ty^l}y^t$.

In information geometry on Riemannian manifolds, Amari and Nagaoka [1] proposed the concept of locally dually flat Riemannian metrics. The information geometry may be seen in the research of geometric structure of the probability distributions and can be applied in various fields such as control system and statistical inference. In [13], Shen enhanced the concept of locally dually flatness. A. Tayebi et.al. [17] studied on generalized m-th root metric. Futher, we study locally dually flatness for metric (1.1), that is $\bar{F}=e^{\sigma(x)}\frac{F^{k+1}}{\beta^k}$ and proved following Theorem:

Theorem 1.2. *Suppose $F=\sqrt[m]{A}$ be m-th root metric on subset $U\subset\mathbb{R}^n$, where U is an open and A is irreducible. Let $\bar{F}=e^{\sigma(x)}\frac{F^{k+1}}{\beta^k}$ is generalized Kropina conformal change of m-th root metric. Then $\bar{F}$ is locally dually flat iff there exists $\theta=\theta_l(x)y^l$ such that*

$$A_{x^l}=\frac{1}{3m}\left[(2k+3-m)A_l\theta+A\theta_l+2A_l\sigma_0+\frac{m}{k+1}A\sigma_{0l}+\frac{2m}{k+1}A\sigma_{x^l}\right], \tag{1.4}$$

$$\beta_{x^l}=\frac{(k+1)}{mA}\left(A_0b_l+A_l\beta_0\right)+\frac{1}{2}\beta_{0l}+b_l\sigma_0 \tag{1.5}$$

and

$$\beta_0b_l=0.$$

In [17], A. Tayebi et. al. demonstrated that two metrics reduce to Riemannian metric, if a generalized m-th root metric is conformal to m-th root metric. Further, B. Tiwari and M. Kumar [18] studied Randers change of m-th root metric.

It is well known that every Riemannian metric of constant curvature is locally conformally flat. In Finsler geometry, the Weyl Theorem states that the projective and conformal properties of a Finsler space determine the metric properties uniquely ([8], [12]).

Here, in this paper, we call metric $\bar{F}$ as generalized Kropina conformal change m-th root metric and $(M,\bar{F})$ as Finsler space of generalized Kropina conformal change, throughout the paper.

2. Preliminaries

Let M be n-dimensional C^∞-manifold. The tangent space at $x\in M$ are denoted by T_xM and the tangent bundle of M denoted by $TM:=\bigcup_{x\in M}T_xM$. Every element of TM is of the form (x,y), where $x\in M$ and $y\in T_xM$. Let $TM_0=TM\setminus\{0\}$.

Definition: A metric on M is a function $F:TM\to[0,\infty)$ with the following properties:

(i) F is C^{∞} on TM_0,
(ii) F is positively 1-homogeneous on the fibers of tangent bundle TM and
(iii) the Hessian of $\frac{F^2}{2}$ with components $g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$ is positive definite on TM_0.

The pair $F^n = (M, F)$ is called a Finsler space of dimension n. F is called fundamental function and tensor g with components g_{ij} is called fundamental tensor of Finsler space F^n.
The normalized element l_i and angular metric tensor h_{ij} are defined, respectively as:

$$l_i = \frac{\partial F}{\partial y^i}, \quad h_{ij} = F\frac{\partial^2 F}{\partial y^i \partial y^j}.$$

Locally, geodesics of a metric $F = F(x, y)$ are given by

$$\frac{d^2x^i}{dt^2} + 2G^i(x, \frac{dx^i}{dt}) = 0, \tag{2.1}$$

where $G^i = \frac{1}{4}g^{il}\{[F^2]_{x^k y^l}y^k - [F^2]_{x^l}\}$ are spray coefficient.

A metric $F = F(x, y)$ is projectively flat on $U \subset \mathbb{R}^n$ iff

$$F_{x^t y^l}y^t - F_{x^l} = 0.$$

A metric F on $U \subset \mathbb{R}^n$ is called locally dually flat if it satisfies the following condition

$$(F^2)_{x^t y^l}y^t = 2(F^2)_{x^l}.$$

3. Proof of Theorem 1.1

From equation (2.1), $G^i(x, y) = P(x, y)y^i$, where $P : TU = U \times \mathbb{R}^n \to \mathbb{R}$ is homogeneous (positively) of degree one in y, that is $P(x, \lambda y) = \lambda P(x, y), \lambda > 0$. Here P is projective factor.
To prove Theorem, we use Lemma [3]:

Lemma 3.1. *Let equation $\Phi A^2 + \Psi A + \Theta = 0$ is true, where Φ, Ψ, Θ are polynomials in y and $m \geq 3$. Then $\Phi = \Psi = \Theta = 0$, [16].*

By metric (1.1), we obtain

$$\left[\bar{F}\right]_{x^l} = e^{\sigma(x)}\frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}}\left[\frac{(k+1)}{m}\beta^2 AA_{x^l} - k\beta A^2\beta_{x^l} + \beta^2 A^2\sigma_{x^l}\right]. \tag{3.1}$$

Similarly

$$\left[\bar{F}\right]_{x^t} = e^{\sigma(x)}\frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}}\left[\frac{(k+1)}{m}\beta^2 AA_{x^t} - k\beta A^2\beta_{x^t} + \beta^2 A^2\sigma_{x^t}\right],$$

and

$$\begin{aligned}\left[\bar{F}\right]_{x^t y^l}y^t = e^{\sigma(x)}\frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}}\Bigg[&\frac{(k+1)}{m}\beta^2 AA_{0l} + \left(\frac{k-m+1}{m}\right)\left(\frac{k+1}{m}\right)\beta^2 A_0 A_l \\ &-k\left(\frac{k+1}{m}\right)A\beta\left(A_0 b_l + \frac{(k+1)}{m}A_l\beta_0\right) - k\beta A^2\beta_{0l} + (k+1)kA^2\beta_0 b_l \\ &+\frac{(k+1)}{m}\beta^2 AA_l\sigma_0 - k\beta A^2 b_l\sigma_0 + \beta^2 A^2\sigma_{0l}\Bigg].\end{aligned} \tag{3.2}$$

Suppose $\bar{F}$ is locally projectively flat. Then we obtain

$$\left[\bar{F}\right]_{x^t y^l}y^t - \left[\bar{F}\right]_{x^l} = 0. \tag{3.3}$$

Therefore, from equations (3.1), (3.2) and (3.3), we obtain

$$\begin{aligned}
&[\bar{F}]_{x^t y^l} y^t - [\bar{F}]_{x^l} = e^{\sigma(x)} \frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}} \left[\frac{(k+1)}{m} \beta^2 A A_{0l} + \left(\frac{k-m+1}{m} \right) \left(\frac{k+1}{m} \right) \beta^2 A_0 A_l \right. \\
&-k \left(\frac{k+1}{m} \right) A\beta \left(A_0 b_l + \frac{(k+1)}{m} A_l \beta_0 \right) - k\beta A^2 \beta_{0l} + k(k+1) A^2 \beta_0 b_l \\
&\left. + \frac{(k+1)}{m} \beta^2 A A_l \sigma_0 - k\beta A^2 b_l \sigma_0 + \beta^2 A^2 \sigma_{0l} \right] \\
&-e^{\sigma(x)} \frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}} \left[\frac{(k+1)}{m} \beta^2 A A_{x^l} - k\beta A^2 \beta_{x^l} + \beta^2 A^2 \sigma_{x^l} \right] = 0.
\end{aligned} \tag{3.4}$$

The equation (3.4) may be written as

$$\begin{aligned}
&e^{\sigma(x)} \frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}} \left[\frac{(k+1)}{m} \beta^2 A A_{0l} + \left(\frac{k+1}{m} \right) \left(\frac{k-m+1}{m} \right) \beta^2 A_0 A_l \right. \\
&-k \left(\frac{k+1}{m} \right) A\beta \left(A_0 b_l + \frac{(k+1)}{m} A_l \beta_0 \right) - k\beta A^2 \beta_{0l} + (k+1) k A^2 \beta_0 b_l \\
&+ \frac{(k+1)}{m} \beta^2 A A_l \sigma_0 - k\beta A^2 b_l \sigma_0 + \beta^2 A^2 \sigma_{0l} \\
&\left. - \frac{(k+1)}{m} \beta^2 A A_{x^l} + k\beta A^2 \beta_{x^l} - \beta^2 A^2 \sigma_{x^l} \right] = 0.
\end{aligned}$$

Or

$$\begin{aligned}
&e^{\sigma(x)} \frac{A^{\frac{k+1}{m}-2}}{\beta^{k+2}} \left[\frac{(k+1)}{m} \beta^2 \left\{ A A_{0l} + \left(\frac{k-m+1}{m} \right) A_0 A_l + A A_l \sigma_0 - A A_{x^l} \right. \right. \\
&\left. + \frac{m}{(k+1)} A^2 \sigma_{0l} - \frac{m}{(k+1)} A^2 \sigma_{x^l} \right\} + A\beta \left\{ -k \left(\frac{k+1}{m} \right) \left(A_0 b_l + \frac{(k+1)}{m} A_l \beta_0 \right) \right. \\
&\left. \left. -kA\beta_{0l} - kAb_l\sigma_0 + kA\beta_{x^l} \right\} + k(k+1) A^2 \beta_0 b_l \right] = 0.
\end{aligned}$$

Using Lemma 3.1, we have

$$AA_{0l} + \left(\frac{k-m+1}{m} \right) A_0 A_l + A A_l \sigma_0 - A A_{x^l} + \frac{m}{(k+1)} A^2 \sigma_{0l} - \frac{m}{(k+1)} A^2 \sigma_{x^l} = 0, \tag{3.5}$$

$$-k \left(\frac{k+1}{m} \right) \left(A_0 b_l + \frac{(k+1)}{m} A_l \beta_0 \right) - kA\beta_{0l} - kAb_l\sigma_0 + kA\beta_{x^l} = 0 \tag{3.6}$$

and

$$k(k+1) A^2 \beta_0 b_l = 0. \tag{3.7}$$

Equations (3.5), (3.6) and (3.7) can be written as

$$mA \left(A_{0l} - A_{x^l} + A_l \sigma_0 + \frac{m}{(k+1)} A\sigma_{0l} - \frac{m}{(k+1)} A\sigma_{x^l} \right) = (m-k-1) A_0 A_l, \tag{3.8}$$

$$A_0 b_l + \frac{(k+1)}{m} A_l \beta_0 + \frac{m}{(k+1)} A\beta_{0l} + \frac{m}{(k+1)} Ab_l \sigma_0 = \frac{m}{(k+1)} A\beta_{x^l}. \tag{3.9}$$

Using (3.9), we obtain equation (1.3) and

$$\beta_0 b_l = 0.$$

If A is irreducible, then deg $(A_l) = m-1$ and there exist 1-form $\theta = \theta_l y^l$ as

$$A_0 = 2m\theta A. \tag{3.10}$$

That is

$$A_{x^t} y^t = 2m\theta A. \tag{3.11}$$

Differentiating equation (3.11) with respect to y^l, we get

$$A_{x^t y^l} y^t + A_{x^l} = 2m \left(\theta_l A + A_l \theta \right).$$

This implies

$$A_{0l} = 2m \left(A_l \theta + A\theta_l \right) - A_{x^l}. \tag{3.12}$$

Using equations (3.8), (3.10) and (3.12), we obtain equation (1.2).

4. Proof of Theorem 1.2

From metric (1.1), we obtain

$$\left[\bar{F}^2\right]_{x^l} = e^{2\sigma(x)} \frac{A^{\frac{2k+2}{m}-2}}{\beta^{2k+2}} \left[\frac{(2k+2)}{m}\beta^2 A A_{x^l} - 2kA^2\beta\beta_{x^l} + 2\sigma_{x^l}\beta^2 A^2\right]. \tag{4.1}$$

$$\left[\bar{F}^2\right]_{x^t} = e^{2\sigma(x)} \frac{A^{\frac{2k+2}{m}-2}}{\beta^{2k+2}} \left[\frac{(2k+2)}{m}\beta^2 A A_{x^t} - 2kA^2\beta\beta_{x^t} + 2\sigma_{x^t}\beta^2 A^2\right]$$

and

$$\begin{aligned}\left[\bar{F}^2\right]_{x^t y^l} y^t = e^{2\sigma(x)} \frac{A^{\frac{2k+2}{m}-2}}{\beta^{2k+2}} \Bigg[&\frac{(2k+2)}{m}\beta^2 A A_{0l} + \left(\frac{(2k+2)}{m} - 1\right)\left(\frac{(2k+2)}{m}\right)\times \\ &\beta^2 A_0 A_l - 2k\left(\frac{(2k+2)}{m}\right) A\beta\left(A_0 b_l + A_l\beta_0\right) - 2k\beta A^2\beta_{0l} \\ &+2k(2k+1)A^2\beta_0 b_l + \frac{2(2k+2)}{m} A A_l \beta^2\sigma_0 - 4k\beta A^2 b_l\sigma_0 + 2\beta^2A^2\sigma_{0l}\Bigg].\end{aligned} \tag{4.2}$$

Consider metric $\bar{F}$ is locally dually flat, then

$$\left[\bar{F}^2\right]_{x^t y^l} y^t - 2\left[\bar{F}^2\right]_{x^l} = 0. \tag{4.3}$$

Therefore, from equations (4.1), (4.2) and (4.3), we obtain

$$\begin{aligned}&e^{2\sigma(x)} \frac{A^{\frac{2k+2}{m}-2}}{\beta^{2k+2}} \Bigg[\frac{(2k+2)}{m}\beta^2 A A_{0l} + \left(\frac{(2k+2)}{m} - 1\right)\left(\frac{(2k+2)}{m}\right)\beta^2 A_0 A_l \\ &\quad -2k\left(\frac{(2k+2)}{m}\right) A\beta\left(A_0 b_l + A_l\beta_0\right) - 2k\beta A^2\beta_{0l} + 2k(2k+1)A^2\beta_0 b_l \\ &+\frac{2(2k+2)}{m} A A_l\beta^2\sigma_0 - 4k\beta A^2 b_l\sigma_0 + 2\beta^2 A^2\sigma_{0l} \\ &\quad -2\left\{\frac{(2k+2)}{m}\beta^2 A A_{x^l} - 2kA^2\beta\beta_{x^l} + 2\sigma_{x^l}\beta^2A^2\right\}\Bigg] = 0.\end{aligned} \tag{4.4}$$

The equation (4.4) may be rewritten as

$$\begin{aligned}e^{2\sigma(x)} \frac{A^{\frac{2k+2}{m}-2}}{\beta^{2k+2}} \Bigg[&\frac{(2k+2)}{m}\beta^2\Big\{AA_{0l} + \left(\frac{(2k+2)}{m} - 1\right) A_0A_l + 2AA_l\sigma_0 \\ &+\frac{2m}{(2k+2)}A^2\sigma_{0l} - 2AA_{x^l} - \frac{4m}{(2k+2)}A^2\sigma_{x^l}\Big\} \\ &+A\beta\Big\{-\frac{2k(2k+2)}{m}\left(A_0b_l + A_l\beta_0\right) - 2kA\beta_{0l} \\ &-4kAb_l\sigma_0 + 4kA\beta_{x^l}\Big\} + 2k(2k+1)A^2\beta_0 b_l\Bigg] = 0.\end{aligned}$$

Using Lemma 3.1, we have

$$\begin{aligned}&AA_{0l} + \left(\frac{(2k+2)}{m} - 1\right) A_0A_l + 2AA_l\sigma_0 + \frac{2m}{(2k+2)}A^2\sigma_{0l} \\ &-2AA_{x^l} - \frac{4m}{(2k+2)}A^2\sigma_{x^l} = 0,\end{aligned} \tag{4.5}$$

$$-\frac{2k(2k+2)}{m}\left(A_0b_l + A_l\beta_0\right) - 2kA\beta_{0l} - 4kAb_l\sigma_0 + 4kA\beta_{x^l} = 0 \tag{4.6}$$

and

$$2k(2k+1)A^2\beta_0 b_l = 0. \tag{4.7}$$

Equations (4.5), (4.6) and (4.7) can be written as

$$\left(2A_{x^l} - A_{0l} - 2A_l\sigma_0 - \frac{2m}{(2k+2)}A\sigma_{0l} + \frac{4m}{(2k+2)}A\sigma_{x^l}\right) = \left(\frac{(2k+2)}{m} - 1\right)A_lA_0, \tag{4.8}$$

$$(2k+2)\left(A_0b_l + A_l\beta_0\right) + mA\beta_{0l} + 2mAb_l\sigma_0 - 2mA\beta_{x^l} = 0. \tag{4.9}$$

Using (4.9), we obtained equation (1.5) and

$$\beta_0 b_l = 0.$$

If A is irreducible, then deg $(A_l) = m-1$ and there exist 1-form $\theta = \theta_l y^l$ as

$$A_0 = \theta A. \tag{4.10}$$

That is

$$A_{x^t}y^t = \theta A. \tag{4.11}$$

Differentiating equation (4.11) with respect to y^l, we get

$$A_{x^t y^l}y^t + A_{x^l} = \theta_l A + A_l\theta.$$

This implies

$$A_{0l} = A_l\theta + A\theta_l - A_{x^l}. \tag{4.12}$$

Using equations (4.8), (4.10) and (4.12), we obtain equation (1.4).

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