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Original Article

A linear-time algorithm to compute total $[1,2]$-domination number of block graphs

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ABSTRACT: Let $G = (V, E)$ be a simple graph without isolated vertices. A set $D \subset V$ is a total $[1,2]$-dominating set if for every vertex $v \in V$, $1 \leq |N(v) \cap D| \leq 2$. The total $[1,2]$-domination problem is to determine the total $[1,2]$-domination number $\gamma_{t[1,2]}(G)$, which is the minimum cardinality of a total $[1,2]$-dominating set for a graph G. In this paper, we present a linear-time algorithm to compute $\gamma_{t[1,2]}(G)$ for a block graph G.

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1. Introduction

All graphs considered here are simple, i.e., finite, undirected, and loop-less. For other graph theory terminology and notation not given here we refer to [10].

Let $G = (V, E)$ be a graph. The *open neighborhood* of a vertex $v \in V$ is the set of all vertices adjacent to v and is denoted by $N(v)$. Similarly, the *closed neighborhood* of a vertex v is $N[v] = N(v) \cup \{v\}$. In connected graph G, a vertex is called a *cut-vertex* of G if its removal produses a disconnected graph. A *block of a graph G* is a maximal connected induced subgraph of G that has no cut-vertex. A block graph is a graph whose blocks are complete graphs. A subset $D \subseteq V$ is called a *dominating set*, if every vertex in V is contained in D or has a neighbor in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set. A *total dominating set* of a simple graph $G = (V, E)$ without isolated vertex, is a set $S \subseteq V$ such that every vertex is adjacent to a vertex in S. The minimum cardinality of a total dominating set is denoted by $\gamma_t(G)$. The minimum dominating set problem is an NP-hard problem [7, 8, 9].

A set $S \subseteq V$ is called a $[1,2]$-*set* of G if for each $v \in V - S$, v is adjacent to at least one but not more than two vertices in S. The total $[1,2]$-set is a set $S \subseteq V$ such that for each $v \in V$, $1 \leq |N(v) \cap S| \leq 2$. The total $[1,2]$-domination number, denoted by $\gamma_{t[1,2]}(G)$, is the minimum cardinality of a total $[1,2]$-dominating set for a graph G. We note that in the problem total $[1,2]$-dominating set, there are some graphs without any total $[1,2]$-dominating sets, such as the graphs with isolated vertices.

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The concept of $[1,2]$-set and its variants such as total $[1,2]$-set and independent $[1,2]$-set are well studied by Chellali et al. in [3, 4]. In [4], several open problems were proposed about $[1,2]$-set and total $[1,2]$-set. Some of these problems were solved in [6, 2, 5]. In this paper we provide a linear-time algorithm for finding the minimum cardinality of a total $[1,2]$-dominating set for a block graph G.

2. Algorithm for computing total $[1,2]$-domination number

Our algorithm relies on a tree-like decomposition structure, which is called a refined cut-tree of a block graph.

Let G be a block graph with t blocks $B_1, B_2, \cdots, B_t$ and q cut-vertices $v_1, v_2, \cdots, v_q$. The cut-tree of G, denoted by $T^C(V^C, E^C)$, is defined as $V^C = \{B_1, B_2, \cdots, B_t, v_1, v_2, \cdots, v_q\}$ and $E^C = \{(B_i, v_j) \mid v_j \in B_i, 1 \leq i \leq h, 1 \leq j \leq q\}$. The cut-tree of a block graph can be constructed in linear-time by the depth-first search algorithm [1]. For any block B_i of G, the block-vertex $\tilde{B}_i$ is defined as $\tilde{B}_i = \{v \in B_i \mid v \text{ is not a cut-vertex}\}$, where $1 \leq i \leq t$. We can refine the cut-tree $T^C(V^C, E^C)$ as $V^C = \{\tilde{B}_1, \cdots, \tilde{B}_t, v_1, \cdots, v_q\}$ and $E^C = \{(\tilde{B}_i, v_j) \mid v_j \in B_i, 1 \leq i \leq t, 1 \leq j \leq q\}$. We notice that in the refined cut-tree of a block graph, a block-vertex can be empty.

A block graph G with 11 blocks $B_1, B_2, \cdots, B_{11}$ and the corresponding refined cut-tree of G are shown in Figure 1.

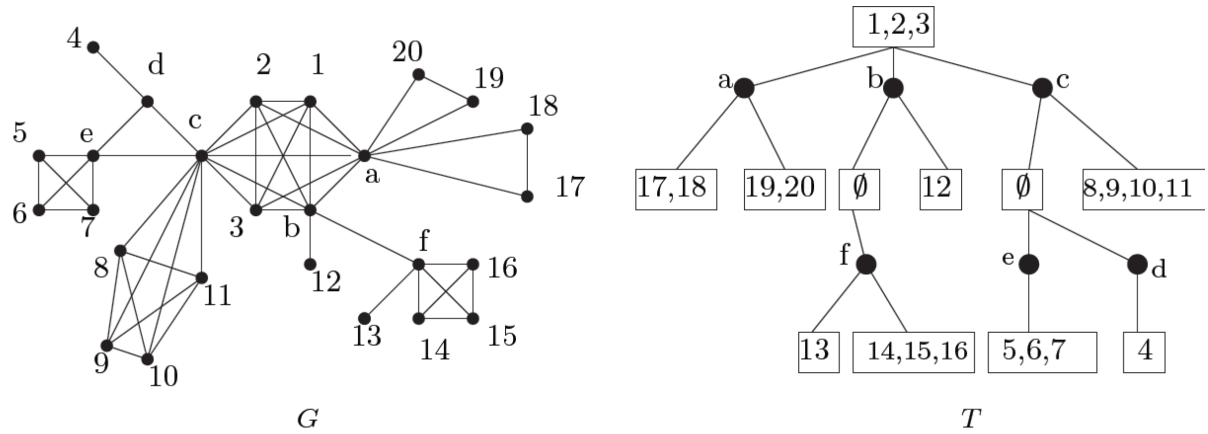

Figure 1: Block graph G and the corresponding cut-tree of G

To compute $\gamma_{[1,2]}(G)$, we traverse T in the post order and during traversing, we compute $m_i^-[v]$, $m_i^+[v]$, $S_0^-(u)$, $S_{0,1}^-(u)$, $S_{1,2}^-(u)$, $S_0^+(u)$, $S_{0,1}^+(u)$ and $S^*(u)$ where $i = 0, 1, 2$, v is cut-vertex and $u \in V(T)$ is block node.

- B as the set of all block nodes of T.
- C as the set of all cut-vertices of G.
- T_v as the subtree of T rooted at v.
- $G[T_v]$ as the subgraph of G which corresponding to T_v.
- For the smallest $[1,2]$-set S of T_v, every vertex in S is called (S, v)-black, and (S, v)-white otherwise. For simplicity, we use the terms black and white instead of (S, v)-black and (S, v)-white, respectively.
- For each block node $u \in V(T)$ and $v \in ch(u)$, depended on the color of v and the number of vertices which dominate v, we define variables as bellow:
  - For $u \in V(T)$, $S_0^-(u)$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that all children of u are white and they are not dominated.
  - For $u \in V(T)$, $S_{0,1}^-(u)$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that all children of u are white and they are dominated at most once.
  - For $u \in V(T)$, $S_{1,2}^-(u)$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that all children of u are white and they are dominated once or twice.
  - For $u \in V(T)$, $S_0^+(u)$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that one child of u is black and it is dominated at most once. Moreover the other children of u are white and they are not dominated.

  - For $u \in V(T)$, $S^+_{0,1}(u)$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that one vertex of $ch(u)$ is black and it is dominated once or twice. Moreover the other children of u are white and they are dominated at most once.
  - For $u \in V(T)$, $S^*(u)$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that two black vertices of $ch(u)$ are dominated at most once. Moreover the other children of u are white and they are not dominated.

- For cut-vertex $v \in V(T)$, $m^-_i[v]$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that v is white and dominated by i other vertices of $G[T_v]$ for $i = 0, 1, 2$.

- For cut-vertex $v \in V(T)$, $m^+_i[v]$ is the size of the smallest $[1,2]$-set of $G[T_v]$ that v is black and dominated by i other vertices of $G[T_v]$ for $i = 0, 1, 2$.

Now, we use a refined cut tree T of a given block graph G and dynamic programming method to compute total $[1,2]$-domination number of G. The algorithm contains three step, Initializing step, Updating step and final step.

2.1. Initializing step:

Obviously, every leaf of cut-tree T is a block node and it is not empty. Since variables for block nodes are based on color of its child, so in first step we begin our algorithm from pre-pendent node v of T, that is a cut vertex of G. We initialize $m^+_i[v]$ and $m^-_i[v]$ for $i = 0, 1, 2$ and pre-pendent node v of T as bellow:

$$m^+_0[v] = 1, \;\; m^+_1 = 2, \;\; m^+_2 = 3, \;\; m^-_0[v] = \infty,$$

$$m^-_1[v] = \begin{cases} 1 & \text{if } |ch(v)| = 1, \\ \infty & \text{Otherwise,} \end{cases} \quad \text{and} \quad m^-_2[v] = \begin{cases} 2 & \text{if } |ch(v)| = 2, \\ \infty & \text{Otherwise.} \end{cases}$$

2.2. Updating step:

In the post order traversal of T, for each non pre-pendent node, based on type of them which are a block or cut, we can consider the following cases:

2.3. Updating step for block nodes of T:

In this step, we define variables $S^-_0(u), S^-_{0,1}(u), S^-_{1,2}(u), S^+_0(u), S^+_{0,1}(u)$ and $S^*(u)$ for block node u of T. These variables depend on the number of nodes in $ch(u)$.

Calculating $S^-_0(u)$:

All children of u are white and they are not dominated. So:

$$S^-_0(u) = \sum_{v \in ch(u)} m^-_0(v).$$

Calculating $S^-_{0,1}(u)$:

All children of u are white and they are dominated at most once. So:

$$S^-_{0,1}(u) = \sum_{v \in ch(u)} Min\{m^-_0(v), m^-_1(v)\}.$$

Calculating $S^-_{1,2}(u)$:

All children of u are white and they are dominated at most once. So:

$$S^-_{1,2}(u) = \sum_{v \in ch(u)} Min\{m^-_1(v), m^-_2(v)\}.$$

Calculating $S^+_0(u)$:

One child $v_i \in ch(u)$ is black, it is dominated at most once and the other children of u are white and they are not dominated. So:

$$S^+_0(u) = Min_{v_i \in ch(u)}\{Min\{m^+_0(v_i), m^+_1(v_i)\} + \sum_{v \in ch(u), v \neq v_i} m^-_0(v)\}.$$

Calculating $S^+_{0,1}(u)$:

One child $v_i \in ch(u)$ is black and it is dominated once or twice. Moreover the other children of u are white and they are dominated at most once. So:

$$S^+_{0,1}(u) = Min_{v_i \in ch(u)}\{Min\{m^+_1(v_i), m^+_2(v_i)\} + \sum_{v \in ch(u), v \neq v_i} Min\{m^-_0(v_i), m^-_1(v_i)\}\}.$$

Calculating $S^*(u)$:

Two vertices of $v_i, v_{i'} \in ch(u)$ are black and they are dominated at most once. Moreover the other children of u are white and they are not dominated. So:

$$S^*(u) = Min_{v_i, v_{i'} \in ch(u)}\{Min\{m^+_0(v_i), m^+_1(v_i)\} + \{Min\{m^+_0(v_{i'}), m^+_1(v_{i'})\} + \sum_{v \in ch(u), v \neq v_i, v_{i'}} m^-_0(v_i)\}.$$

2.4. Updating step for cut vertex of T:

In this step, for $i = 0, 1, 2$ we define variables $m^-_0 i(v)$ and $m^+_i(v)$ for cut nodes v of T.

Calculating $m^+_0(v)$ when v is not a pre-pendent:

In this case, v is black so all the children of v and all children of $ch(v)$ already have a black neighbor. Since v should not dominate by any vertices, So, all child $u \in ch(v)$ must be white and they are dominated at most once.

$$m^+_0(v) = 1 + \sum_{u \in ch(v)} S^-_{0,1}(u).$$

Calculating $m^+_1(v)$ when v is not a pre-pendent:

In this case, v is black and it is dominated once, so one of the following cases can occurs:

- In this case, exactly one vertex of block $u_i \in ch(v)$ is black and other children of u_i have color white and are not dominated. For the other block $u \in ch(v)$, all child are white and they are dominated at most once. We have:

$$M^+_1 = 1 + Min_{u_i \in ch(v), |u_i| \neq 0}\{S^-_0(u_i) + \sum_{u \in ch(v), u \neq u_i} S^-_{0,1}(u)\}.$$

- For exactly one block $u_i \in ch(v)$, one nodes of $ch(u_i)$ and the black vertex dominate at most once. The other vertices of $ch(u_i)$ are white and they are not dominated. In addition, for the other block $u \in ch(v)$, all of their children are white and they are dominated at most once. We have:

$$M'^+_1 = Min_{u_i \in ch(v)}\{S^+_0(u_i) + \sum_{u \in ch(v), u \neq u_i} S^-_{0,1}(u)\}.$$

Minimum of M^+_1 and M'^+_1 is the best value for $m^+_1(v)$. So:

$$m^+_1(v) = Min\{M^+_1, M'^+_1\}.$$

Calculating $m^+_2(v)$ when v is not a pre-pendent:

In this case, v is black and it is dominated twice, so one of the following cases can occurs:

- There exist at least one vertex $u_i \in ch(v)$ such that $|u_i| = |ch(u_i)| = 1$. In this case, vertex in block u_i is black, its child is black and it is not dominated. So we have:

$$M^+_2 = 1 + Min_{u_i \in ch(v), |u_i| = |ch(u_i)| = 1}\{m^+_0(ch(u_i))\} + \sum_{u \in ch(v), u \neq u_i} S^-_{0,1}(u).$$

  If there is not any node $u_i \in ch(v)$ such that $|u_i| = |ch(u_i)| = 1$, then $M^+_2 = \infty$.

- There exist at least one vertex $u_i \in ch(v)$ such that $|u_i| = 0$ and $|ch(u_i)| = 2$. In this case, both children of $ch(u_i)$ are black and it is not dominated. So we have:

$$M'^+_2 = Min_{u_i \in ch(v), |u_i| = 0, |ch(u_i)| = 2}\{m^+_0(ch(u_i))\} + \sum_{u \in ch(v), u \neq u_i} S^-_{0,1}(u).$$

  If there is not any node $u_i \in ch(v)$ such that $|u_i| = |ch(u_i)| = 1$, then $M'^+_2 = \infty$.

- For exactly two block $u_i, u_j \in ch(v)$, one vertex is black, so all children of $ch(u_i)$ and $ch(u_j)$ should be white and they are not dominated. For the other block $u \in ch(v)$, all child must be white and they are dominated at most once. We have:

$$M_2^{''+} = 2 + Min_{u_i,u_j \in ch(v)}\{S_0^-(u_i) + S_0^-(u_j) + \sum_{u \in ch(v), u \neq u_i,u_j} S_{0,1}^-(u)\}.$$

- For exactly one block $u_i \in ch(v)$, one vertex is black, so all children of $ch(u_i)$ should be white and they are not dominated. In addition, for exactly one block $u_j \in ch(v)$, exactly one child is black and dominated at most once and the other child is white and not dominated. Also, all children other child $u \in ch(v)$ should be white and they are dominated at most once.. We have:

$$M_2^{'''+} = 1 + Min_{u_i,u_j \in ch(v)}\{S_0^-(u_i) + S_0^+(u_j) + \sum_{u \in ch(v), u \neq u_i,u_j} S_{0,1}^-(u)\}.$$

- For exactly two blocks $u_i, u_j \in ch(v)$, exactly one child is black and dominated at most once and the other child is white and not dominated. Also, all children other child $u \in ch(v)$ should be white and they are dominated at most once.. We have:

$$M_2^{''''+} = Min_{u_i,u_j \in ch(v)}\{S_0^-(u_i) + S_0^-(u_j) + \sum_{u \in ch(v), u \neq u_i,u_j} S_{0,1}^-(u)\}.$$

Minimum of $M_2^{'+}, M_2^{''+}, M_2^{'''+}$ and $M_2^{''''+}$ is the best value for $m_1^+(v)$. So:

$$m_1^+(v) = Min\{M_2^{'+}, M_2^{''+}, M_2^{'''+}, M_2^{''''+}\}.$$

Calculating $m_0^-(v)$ when v is not a pre-pendent

In this case, v is white and none of the child of v and $ch(v)$ are not black. If v has a child like u that is a none empty block node, then vertices of u can not dominated by any vertices because v or $ch(u)$ can only dominate u. So, $m_0^-(v) = \infty$ otherwise we have:

$$m_1^-(v) = \sum_{u \in ch(v)} S_{1,2}^-(u).$$

Calculating $m_1^-[v]$ when v is not a pre-pendent

In this case, v is white and exactly one of the child of v or $ch(v)$ are black.

- The node v has at least two children like u_1 and u_2 that are none empty block node. So vertices of u_1 or vertices of u_2 can not dominated and $m_1^-(v) = \infty$.
- The node v has only one none empty child like u_1 and the other children are empty. So two cases appear:
  1. One of the vertex of block node u_1 is black and all of children of u_1 are white and are dominated at most once.
  2. All vertices of block node u_1 are white, exactly one child v_i of u_1 is black, the other are white. Moreover, v_i is dominated once or twice and white siblings of v_i are dominated at most once.

  And we have:

$$m_1^-(v) = Min\{1 + S_{0,1}^-(u_1) + \sum_{u \in ch(v), u \neq u_1} S_{1,2}^-(u), S_{0,1}^+(u) + \sum_{u \in ch(v), u \neq u_1} S_{1,2}^-(u)\}$$

- All children of v are empty. So, among all children of v, there is exactly one node u_i such that u_i has only a black child the other is white. In other child $u \neq u_i$, all nodes of $ch(u)$ should be white and we have:

$$m_1^-(v) = Min_{u_i \in ch(v)}\{S_{0,1}^+(u_i) + \sum_{u \in ch(v), u \neq u_i} S_{1,2}^-(u)\}$$

  All children of v are empty. So, from exactly one child u_i of v, one of the children is black and the others are white. In other child $u \neq u_i$, all nodes of $ch(u)$ should be white and we have:

$$m_1^-(v) = Min_{u_i \in ch(v)}\{S_{0,1}^+(u_i) + \sum_{u \in ch(v), u \neq u_i} S_{1,2}^-(u)\}$$

Calculating $m_2^-[v]$ when v is not a pre-pendent

In the last cases, we consider v is white and exactly two children of v or $ch(v)$ are black.

- The node v has more than two none empty children, $m_2^-[v] = \infty$.
- The node v has only two none empty children like u_1 and u_2 and the other children are empty. So, for u_1, u_2 and the other vertices one of the following cases appears:
  1. One of the vertex of block nodes u_1 and u_2 are black, all of children of them are white and are dominated at most once.
  2. One of the vertex of block node u_1 is black, all of children of it are white and are dominated at most once. Moreover, all of the vertices of block node u_2 are white, one of its child is black, the others are white and dominated at most once. (u_1 and u_2 can replace.)
  3. All of the vertices of block node u_1, u_2 are white, one of its child is black, the others are white and dominated at most once.

  So we have:

$$\begin{aligned} m_2^-(v) = Min\{&2 + S_{0,1}^-(u_1) + S_{0,1}^-(u_2) + \sum_{u \in ch(v), u \neq u_1, u_2} S_{1,2}^-, \\ &1 + S_{0,1}^+(u_1) + S_{0,1}^-(u_2) + \sum_{u \in ch(v), u \neq u_1, u_2} S_{1,2}^-, \\ &1 + S_{0,1}^+(u_2) + S_{0,1}^-(u_1) + \sum_{u \in ch(v), u \neq u_1, u_2} S_{1,2}^-, \\ &S_{0,1}^+(u_1) + S_{0,1}^+(u_2) + \sum_{u \in ch(v), u \neq u_1, u_2} S_{1,2}^-\} \end{aligned}$$

- The node v has only one none empty child u_1. So, one of the following cases occurs for u_1:
  1. Two children of u_1 are black and the others are white and not dominated at most once.
  2. One vertex of block node u_1 and one of its child are black and the other children of u_1 are white and not dominated.
  3. One vertex of block node u_1 is black and all of children of u_1 are white and are dominated at most once.
  4. All vertices of block node u_1 are withe, one of its child are black and the other children of u_1 are white and not dominated.

  For other child u of $ch(v)$, there are exactly one node u_i such that one child of it is black and the others are white. Obviously, all child of other siblings u_i are white. So we have:

$$\begin{aligned} m_2^-(v) = Min\{&S^*(u_1) + \sum_{u \in ch(v), u \neq u_1} S_{1,2}^-(u_1), \\ &1 + S_0^+(u_1) + \sum_{u \in ch(v), u \neq u_1} S_{1,2}^-(u_1), \\ &1 + S_{0,1}^-(u_1) + Min_{u_i \in ch(v), u_i \neq u_1}\{S_{0,1}^+(u_i) + \sum_{u \in ch(v), u \neq u_i, u_1} S_{1,2}^-(u)\}, \\ &S_{0,1}^+(u_1) + Min_{u_i \in ch(v), u_i \neq u_1}\{S_{0,1}^+(u_i) + \sum_{u \in ch(v), u \neq u_i, u_1} S_{1,2}^-(u)\}\}. \end{aligned}$$

- All children of v are empty. So, from exactly two children u_i, u_j of v, one of the children is black and the others are white. In other child $u \neq u_i, u_j$, all nodes of $ch(u)$ should be white and we have:

$$m_2^-(v) = Min_{u_i, u_j \in ch(v)}\{S_{0,1}^+(u_i) + S_{0,1}^+(u_j) + \sum_{u \in ch(v), u \neq u_i, u_j} S_{1,2}^-(u)\}$$

2.5. Final state:

Let r be the root of refined cut tree T, r can be correspond to a cut vertex of G or a block of it. Depend on type of r one of the following cases appear:

1. The root r of T is a cut vertex of G.
   Since for $i = 0, 1, 2$ we compute $m_i^+[v]$ and $m_i^-[v]$ on a node of T that its corresponding vertex in G is a cut vertex, so we must choose best set among computed set of root r. Note that r should be black or white and should be dominated by one or two vertices. It means that:
   $$M = Min\{m_1^+(r), m_2^+(r), m_1^-(r), m_2^-(r)\}.$$
2. The root r of T is corresponding to a block of G.
   Since, we computed $m^+[v], m_0^-[v], m_1^-[v]$ and $m_2^-[v]$ for all vertices $v \in ch(r)$. Based on the number of vertices in block r and the number of its child, one of the following cases appear:
   (a) $|r| = 0$, so we have:
   $$M = Min\{S_{1,2}^-(r), S_{0,1}^+(r), S^*(r)\}.$$
   (b) $|r| > 0$, so we have::
   $$M = Min\{1 + S_0^+(r), 1 + S_{0,1}^-(r), S_{0,1}^+(r), S_{1,2}^-(r), S^*(r)\}.$$

Theorem 2.1. *The value M computed by Algorithm 1 for the block graph G is size of the smallest total $[1, 2]$-set of G and is computed in linear-time.*

Proof. The number of nodes in the refined cut tree T corresponding to block graph G is linear based on order of G, i.e. n. It is obvious that the algorithm traverses T once and is computed in linear-time $O(n)$. □

Algorithm 1 Total $[1, 2]$-Dominating Set

Input: A refined cut tree T of block graph G.

procedure Initializing step

procedure Updating step: ▷ Depending on the type of non pre-pendant nodes in the post order traversal of T, one of the following procedures is selected:

  procedure Updating step for block nodes of T:

    Calculating $S_0^-(u)$, $S_0^-(u)$, $S_{0,1}^-(u)$, $S_{1,2}^-(u)$, $S_0^+(u)$, $S_{0,1}^+(u)$ and $S^*(u)$.

  procedure Updating step for cut vertex of T

    Calculating $m_0^+(v)$, $m_1^+(v)$, $m_0^-(v)$, $m_1^-[v]$ and $m_2^-[v]$ when v is not a pre-pendent.

procedure Final State

  if the root r of T is a cut vertex of G **then** $M = Min\{m_1^+(r), m_2^+(r), m_1^-(r), m_2^-(r)\}$.

  if the root r is a block and $|r| = 0$ **then** $M = Min\{S_{1,2}^-(r), S_{0,1}^+(r), S^*(r)\}$.

  if the root r is a block and $|r| > 0$ **then**

$$M = Min\{1 + S_0^+(r), 1 + S_{0,1}^-(r), S_{0,1}^+(r), S_{1,2}^-(r), S^*(r)\}.$$

Output: Size of minimum total $[1, 2]$-set of G.

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